2024 Tenth Indian Control Conference (ICC)
9 - 11 December, 2024
IISER Bhopal, India

Data-driven reference tracking of a flexible link rotary servo system

Surya Narayan Santra and Mousumi Mukherjee

***Abstract*— We consider the problem of designing and implementing feedback controllers, based on data, for a lab-scale rotary flexible link system. We first consider the data-driven stabilization problem and show that while the system can be stabilized using a data-driven state-feedback controller, the reference tracking with this controller is not satisfactory. For tracking a rotary servo angle reference, we then design a data-driven reference tracking controller. We provide necessary and sufficient design conditions, in terms of data-based linear matrix inequalities. The controllers are also implemented in real-time.**

***Index Terms*— data-driven control, rotary flexible link, linear matrix inequalities.**

I. Introduction

The model-based approach of analysis and control of dynamical systems consist of deriving a mathematical model of the system either from first principles or by using system identification to *identify* a model of the system. The model is then used for analysis and subsequently for controller synthesis. The model-based approach, albeit successful, has some limitations; for example, deriving a model for a complex, interconnected, and large scale system may be difficult and/or expensive.

In this work, we focus on the *direct data-driven approach*, where controllers are designed directly from measured data without invoking the system matrices explicitly. In particular, we use the data-driven method inspired by the so-called *fundamental lemma* of Willems' for state-space systems [1]. The fundamental lemma [2], originally developed for system in the behavioral framework, has seen renewed interest in the data-driven paradigm in recent years; see [3] for an up-to-date review.

The main objectives of this work are to design: (i) a data-driven stabilizing state feedback controller, and (ii) a data-driven reference tracking controller for a laboratory scale flexible link rotary servo workstation. The flexible link rotary servo workstation emulates various real-world applications, such as, lightweight robot manipulators used in space applications and cantilever beams. In comparison to rigid manipulator systems, flexible manipulators offer several advantages, such as, faster response, lower energy consumption, and lower overall weight. However, the control of flexible systems is not straightforward [4], [5], [6], [7]. Therefore, we believe that data-driven approaches in conjunction with traditional approaches can provide a better understanding of controlling such complex systems.

The organization of the paper is as follows. After describing the system briefly, we state the problem in Section II. The data-driven stabilization, and reference tracking are discussed in Sections III, and IV, respectively. We conclude the paper in Section V.

The authors are with the Department of Electrical Engineering, Indian Institute of Engineering Science and Technology, Shibpur, West Bengal, India. Email: suryagate2022@gmail.com, mousumi@ee.iiests.ac.in.

This work was supported in part by the DST-INSPIRE Faculty Grant (Faculty Reg. No.: IFA22-ENG 348), the Department of Science and Technology (DST), Government of India.

II. System description and Problem statement

A. System description

The flexible link rotary servo workstation by Quanser [8] consists of a rotary servo base unit and a flexible link as shown in Fig. 1.

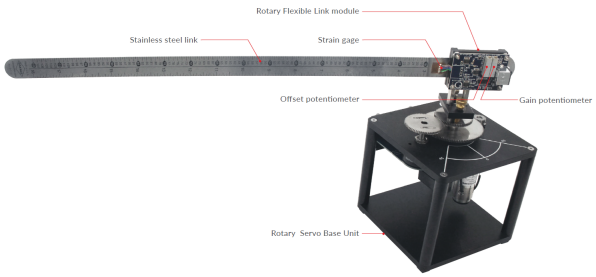


Fig. 1. Flexible link rotary servo workstation (Pic courtesy: Quanser [8])

A DC motor on the rotary servo base unit is used to rotate the flexible link in the horizontal plane. The end of the link connected to the motor is fitted with a strain gage to detect the deflection of the tip. The rotary servo base angle, measured using an incremental encoder, is denoted by θ, and the deflection of the flexible link, relative to the servo, measured using a strain gage, is denoted by α. A schematic diagram depicting these angles is shown in Fig. 2.

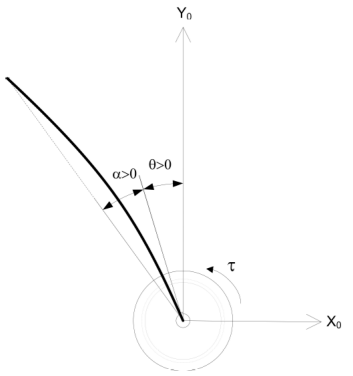


Fig. 2. A schematic diagram of the angles (Pic courtesy: Quanser [8])

A linear state-space model derived for this system is the following [8]

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{K_s}{J_{eq}} & -\frac{B_{eq}}{J_{eq}} & 0\\ 0 & -\frac{K_s(J_l+J_{eq})}{J_l J_{eq}} & \frac{B_{eq}}{J_{eq}} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \frac{1}{J_{eq}}\\ -\frac{1}{J_{eq}}\end{bmatrix}u \tag{1a}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 0\end{bmatrix}u, \tag{1b}$$

where $x_1 = \theta$, $x_2 = \alpha$, $x_3 = \dot{\theta}$, $x_4 = \dot{\alpha}$, K_s is the stiffness of the spring, B_{eq} is the viscous friction coefficient of the rotary servo, J_{eq} is the total moment of inertia of the rotary servo, J_l is the moment of inertia of the flexible link, and τ is the motor torque.

B. Problem statement

For the rotary servo system with flexible link, the objective is to design a controller that will (a) control the position of the servo to a desired angle, and (b) minimize the deflection of the flexible link as the servo position changes. Using the linear state-space model of the system (1), a controller can be designed to achieve the above mentioned objectives.

The focus of this current work, however, is not to use the model of the system. Instead, we measure the state and input data of the system over a finite time interval and utilize this offline data for synthesizing controllers. Note that, the servo angle, θ, that is, x_1, and the link angle α, that is, x_2, are directly measured. The respective velocities, that is, the states x_3 and x_4 are computed by taking the derivatives and filtering the results through a high-pass filter. Thus, for a sampling time $\delta > 0$, the states and input

$$X_- := \begin{bmatrix}x(0) & x(\delta) & \dots & x((T-1)\delta)\end{bmatrix} \tag{2a}$$

$$U_- := \begin{bmatrix}u(0) & u(\delta) & \dots & u((T-1)\delta)\end{bmatrix} \tag{2b}$$

are available. Note that, the data is assumed to be noise-free.

Problem 1: The aim is to design (i) a state feedback controller using (X_-, U_-) such that the closed loop system is stable, and (ii) a data-driven reference tracking controller to track a desired servo position.

III. Data-driven stabilization

The problem of data-driven stabilization for a controllable linear time-invariant (LTI) system in the discrete domain has been thoroughly discussed in [1]. We briefly state some of the results that we require here for the continuous case.

A. General results

An important assumption in establishing the stabilization result [1, Theorem 3] is the following.

Assumption 1: The data (X_-, U_-) are such that

$$\operatorname{rank}\begin{bmatrix}X_-\\ U_-\end{bmatrix} = n + m,$$

where n is the order of the system and m is the number of inputs.

Assumption 1 is equivalent to identifiability of the system. However, the controllers are parametrized using the measured data directly. Further, it can be shown that if the system is controllable and U_- is *persistently exciting* (see [2] for a definition) of order $n+1$, the matrix $\begin{bmatrix}X_-\\ U_-\end{bmatrix}$ is full row rank.

It is also known that if the input data U_- is persistently exciting of order $n + 1$, the closed-loop system, with state feedback $u = -Kx$, can be parametrized using the data matrices (X_-, U_-). We state this result for the continuous case in Theorem 1; see [1, Theorem 2] for the discrete time case.

Theorem 1: Let Assumption 1 hold. Then, a system $\dot{x} = Ax + Bu$, with n states, in closed-loop with state feedback $u = -Kx$ is equivalently given by

$$\dot{x} = X_+ G x, \tag{3}$$

where $X_+ := \begin{bmatrix}\dot{x}(0) & \dot{x}(\delta) & \dots & \dot{x}((T-1)\delta)\end{bmatrix}$, and G is such that

$$\begin{bmatrix}I_n\\ -K\end{bmatrix} = \begin{bmatrix}X_-\\ U_-\end{bmatrix} G, \tag{4}$$

where I_n is the identity matrix of order n. Specifically, the state feedback is given by $u = U_- G x$.

Theorem 1 gives a data-driven parametrization of the closed-loop system. To determine if this closed-loop system is stable or not, we need to compute the eigenvalues of the matrix X_+G. The controller $u(k) = U_- G x(k)$ is said to be a stabilizing state feedback controller if it is such that the eigenvalues of the closed-loop system lie in the left half of the complex plane. Theorem 2 provides a necessary and sufficient condition, in terms of linear matrix inequalities (LMIs), for designing a data-driven state feedback controller that guarantees stability of the closed-loop system. The result follows from a straightforward application of Lyapunov stability theory [9] applied to the data-driven parametrization of the closed-loop system obtained in Theorem 1.

Theorem 2: Let Assumption 1 hold. The system $\dot{x} = Ax + Bu$ is stabilizable using state feedback if and only if there exist a matrix Q such that

$$X_+ Q + Q^\top X_+^\top \prec 0, \tag{5a}$$

$$X_- Q \succ 0 \tag{5b}$$

hold. Furthermore, if (5) is feasible, a stabilizing state feedback gain $K \in \mathbb{R}^{m\times n}$ is given by $K = -U_- Q (X_- Q)^{-1}$.

B. Data-driven stabilization of the flexible link rotary servo system

For the flexible link rotary servo system described in Section II-A, the objective is to use Theorem 2 to design a state feedback controller that stabilizes the system.

In order to do this, first the state data is collected from the real time system in open loop by applying a persistently exciting input. The servo base angle θ and the flexible link angle α are directly obtained from the sensors. The states corresponding to the velocities are obtained by taking derivatives

and filtering the signals through a high pass filter. The measurements were recorded by using a sampling time of $0.002s$. Thus the data matrices $X_- = \begin{bmatrix} x(0) & x(1) & \dots & x(N) \end{bmatrix}$ and $U_- = \begin{bmatrix} u(0) & u(1) & \dots & u(N) \end{bmatrix}$ are formed, where $N = 2500$. The matrix $X_+ = \begin{bmatrix} \dot{x}(0) & \dot{x}(1) & \dots & \dot{x}(N) \end{bmatrix}$ is then constructed using X_- as follows: for $i = 0, 1, \dots, N$, $\dot{x}(i) = \frac{x(i+1)-x(i)}{0.002}$.

Now to design a data-driven stabilizing state feedback controller, the LMIs (5) are solved for feasibility using CVX in MATLAB using the data matrices X_- and X_+. The LMIs are feasible and Q is obtained. Thus, the stabilizing controller K is computed as

$$K = \begin{bmatrix} 0.0716 & -4.7729 & 0.9079 & -0.4019 \end{bmatrix}.$$

The eigenvalues of the closed loop system are $-131.84, -0.05, -5.21 \pm 0.1892i$ which shows that the data-driven controller has indeed stabilized the system. This can be further validated from the experimental plots of θ, and α, as shown in Fig. 3, and Fig. 4, respectively. However, reference tracking with this same K is not satisfactory as shown in Fig. 5.

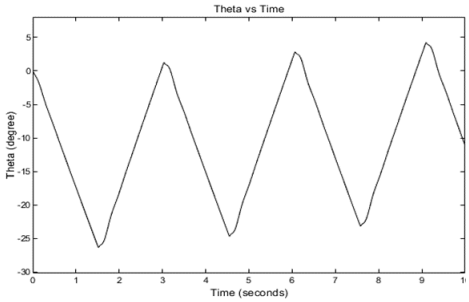


Fig. 3. Real-time variation of (closed-loop) theta

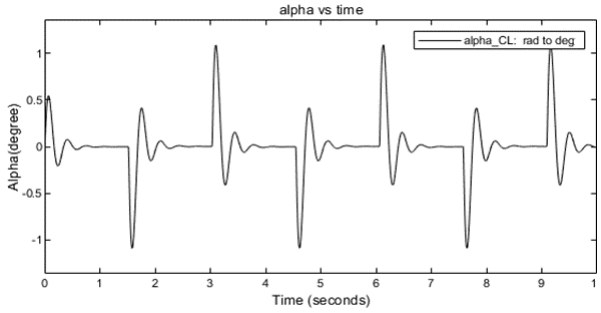


Fig. 4. Real-time variation of (closed-loop) alpha

IV. Reference tracking of the rotary servo flexible link system

As shown in the previous section, a data-driven stabilizing controller, in general, may not be able to track a given reference. Therefore, in this section the objective is to design a data-driven reference tracking controller. We first discuss the model-based case and then go on to discuss the data-driven counterpart.

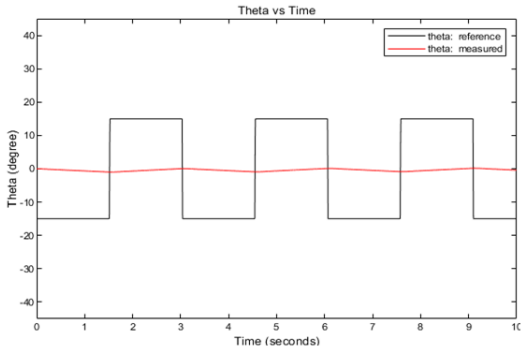


Fig. 5. Poor reference tracking with the data-driven stabilizing controller

A. Model-based case

Consider a continuous linear system in state-space form as

$$\dot{x} = Ax + Bu, \tag{6}$$

where $x \in \mathbb{R}^n$, and $u \in \mathbb{R}^m$. Recall that the rotary servo with a flexible link system has a state-space representation as given in (1a). We now discuss the case of tracking only a reference servo angle θ_{ref}. Let $r := \begin{bmatrix} \theta_{ref} \\ 0 \\ 0 \\ 0 \end{bmatrix}$ be the reference vector. Defining the error signal as $e(t) := r - x(t)$, the open loop system is written as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ \dot{r} - \dot{e} &= A(r - e) + Bu \\ -\dot{e} &= Ar - Ae + Bu. \end{aligned}$$

Since $Ar = 0$ for any reference servo angle θ_{ref}, it follows that the open loop system described in the error variable e is given by

$$\dot{e} = Ae - Bu. \tag{7}$$

Defining the feedback control law to be $u = Ke$, the closed loop error dynamics becomes

$$\dot{e} = (A - BK)e. \tag{8}$$

Thus, to track a reference servo angle, the error must die down to zero exponentially fast, or, in other words, the error dynamics must be exponentially stable. This can be achieved by designing a feedback controller K akin to a state feedback controller when the system matrices A and B are known. In the following section, we discuss the data-driven analogue of the reference tracking controller.

B. Data-driven case

In this section, we discuss the design and implementation of the reference tracking controller using only data. We first note that a data-driven parametrization of the open loop system (7) can be written as

$$E_+ = AE_- - BU_-, \tag{9}$$

where

$$E_- := \begin{bmatrix} e(0) & e(\delta) & \dots & e((T-1)\delta) \end{bmatrix}, \tag{10a}$$

$$U_- := \begin{bmatrix} u(0) & u(\delta) & \dots & u((T-1)\delta) \end{bmatrix}, \text{and} \tag{10b}$$

$$E_+ := \begin{bmatrix} \dot{e}(0) & \dot{e}(\delta) & \dots & \dot{e}((T-1)\delta) \end{bmatrix}. \tag{10c}$$

The data matrix E_- can be easily constructed from the data matrix X_- because the reference is already a known signal. Once E_- is obtained, E_+ can be computed in exactly the same way as X_+ was computed from X_-.

Analogous to Assumption 1, we require the following assumption on the data matrices to obtain a data-driven parametrization of the closed loop system.

Assumption 2: The data (E_-, U_-) are such that

$$\text{rank} \begin{bmatrix} E_- \\ U_- \end{bmatrix} = n + m,$$

where n is the order of the system and m is the number of inputs.

We now state Theorem 3, which gives a data-driven parametrization of the closed loop error dynamics, with feedback $u = Ke$, using the data matrices (E_-, U_-).

Theorem 3: Let Assumption 2 hold. Then, system (7) in closed-loop with feedback $u = Ke$ is equivalently given by

$$\dot{e} = E_+ G e, \tag{11}$$

where G is such that

$$\begin{bmatrix} I_n \\ K \end{bmatrix} = \begin{bmatrix} E_- \\ U_- \end{bmatrix} G, \tag{12}$$

where I_n is the identity matrix of order n. Specifically, the feedback is given by $u = U_- G e$.

Theorem 3 gives a data-driven parametrization of the closed loop error dynamics. For the error to exponentially die down, Theorem 4 provides a necessary and sufficient condition, in terms of LMIs, for designing a data-driven feedback controller that guarantees reference tracking.

Theorem 4: Let Assumption 2 hold. Then, system (7) is stabilizable using feedback if and only if there exist a matrix P such that

$$E_+ P + P^\top E_+^\top \prec 0, \tag{13a}$$

$$E_- P \succ 0 \tag{13b}$$

hold. Furthermore, if (13) is feasible, a feedback gain $K \in \mathbb{R}^{m \times n}$ given by $K = U_- P (X_- P)^{-1}$ ensures reference tracking.

We now apply Theorem 4 to the rotary flexible link system. The data matrices $E_- = \begin{bmatrix} e(0) & \dots & e(N) \end{bmatrix}$ and $U_- = \begin{bmatrix} u(0) & \dots & u(N) \end{bmatrix}$ are formed, where $N = 2500$. The matrix E_+ is then constructed. Using E_- and E_+ the LMIs (13) are checked for feasibility. The LMIs turn out to be feasible and P is obtained. Thus, the data-driven reference tracking controller is computed as

$$K = \begin{bmatrix} 1.9394 & 5.5873 & -0.0896 & -0.3868 \end{bmatrix}.$$

The experimental plot of the rotary servo angle θ for the closed loop system with the data-driven reference tracking controller is shown is Fig 6. The experimental plot for α is shown in Fig. 7.

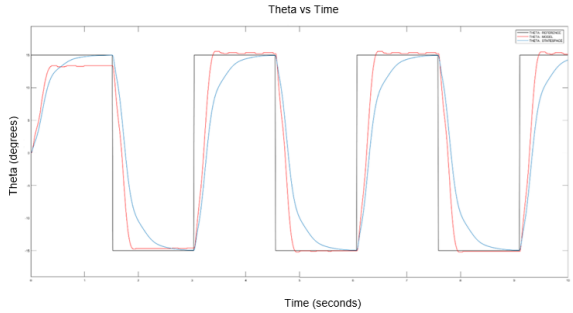


Fig. 6. Variation of theta with the data-driven reference tracking controller in real-time (red curve) compared with simulation (blue curve)

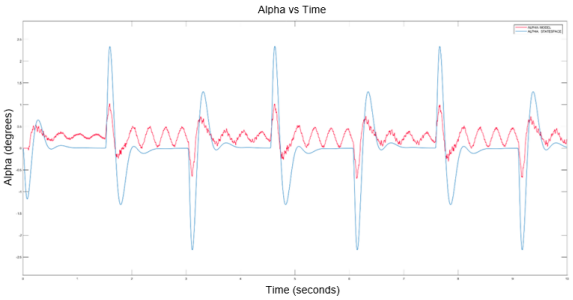


Fig. 7. Variation of alpha with the data-driven reference tracking controller in real-time (red curve) compared with simulation (blue curve)

V. Concluding Remarks

In this article, we designed and implemented direct data-driven controllers, namely, a data-driven stabilizing controller, and a data-driven reference tracking controller to track the servo angle for a lab-scale rotary flexible link system.

Acknowledgement

The authors thank Prof. Aparajita Sengupta for many helpful discussions.

References

[1] C. De Persis and P. Tesi, "Formulas for data-driven control: Stabilization, optimality, and robustness," *IEEE Transactions on Automatic Control*, vol. 65, no. 3, pp. 909–924, 2020.

[2] J. C. Willems, P. Rapisarda, I. Markovsky, and B. L. D. Moor, "A note on persistency of excitation," *Systems & Control Letters*, vol. 54, no. 4, pp. 325–329, 2005.

[3] I. Markovsky and F. Dörfler, "Behavioral systems theory in data-driven analysis, signal processing, and control," *Annual Reviews in Control*, vol. 52, pp. 42–64, 2021.

[4] D. Wang and M. Vidyasagar, "Transfer functions for a single flexible link," *The International Journal of Robotics Research*, vol. 10, no. 5, pp. 540–549, 1991.

[5] M. Moallem, R. V. Patel, and K. Khorasani, "Nonlinear tip-position tracking control of a flexible-link manipulator: theory and experiments," *Automatica*, vol. 37, pp. 1825–1834, 2001.

[6] E. Pereira, S. S. Aphale, V. Feliu, and S. O. R. Moheimani, "Integral resonant control for vibration damping and precise tip-positioning of a single-link flexible manipulator," *IEEE/ASME Transactions on Mechatronics*, vol. 16, no. 2, pp. 232–240, 2011.

[7] A. Shawky, D. Zydek, Y. Z. Elhalwagy, and A. Ordys, "Modeling and nonlinear control of a flexible-link manipulator," *Applied Mathematical Modelling*, vol. 37, pp. 9591–9602, 2013.

[8] J. Apkarian, P. Karam, and M. Levis, *Rotary Flexible link Student Workbook*. Quanser.

[9] H. K. Khalil, *Nonlinear systems; 3rd ed.* Pearson Education Inc., Noida, India: Pearson, 2014.