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Frequency-limited Gramian Framework for Truncated Balanced Realization of Continuous-time Systems

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Abstract— **In this work, a novel configuration of frequency-limited controllability and observability Gramians is constructed to develop a new truncated balanced realization for minimizing the error of higher-order models in specified frequency intervals. The suggested technique generates stable simplified models and approximates better than existing techniques. Two numerical examples are used to assess the performance of the presented approach, and it is found through the error plots and comparison that the proposed reduced models approximate the actual models satisfactorily in the specified frequency band.**

I. INTRODUCTION

In control engineering, mathematical modeling is used for the analysis of a number of physical systems, which results in large-scale models of complex dynamics. The analysis, implementation, and controller design for such systems become challenging due to high order. Therefore, such higher-order systems must be simplified to a simpler system of lower-state space dimension that behaves closely to the actual system. Model Order Reduction (MOR) deals with such issues and provides a Reduced-Order Model (ROM) by retaining the dominant properties of the actual system. The direction of MOR was entirely changed following the development of the truncated balanced realization (TBR) [1] and optimal Hankel norm approximation [2]. Moore [1] introduced the TBR technique using the balanced realization theory, which attracted a lot of interest from researchers. The TBR method produced stable ROMs with a *priori* frequency response bounds [2] and closed-form solutions.

It is found that BT [1] and many other conventional methods [2], [3] minimize the error between the actual and ROMs considering the entire frequency range. It also doesn't need to be mentioned that no actuator or sensor can function across the complete frequency region. The adoption of frequency weights in MOR was motivated by the need for error minimization in some applications over specific frequency ranges. Considering frequency-weights along with TBR, Enns [4] introduced frequency-weighted TBR, which could use input weight, output weight, and both-sided weighting functions. Later, it was pointed out in [5] that the Enns approach [4] results in unstable ROMs for a stable system with double-sided weightings. Therefore, several researchers [6], [7], [8], [9], [10] worked on different algorithms towards the instability issues of [4]. Furthermore, it has been noted that creating frequency weights presents its own set of challenges. Therefore, Gawronski and Juang (GJ) [11] introduced a new area of MOR in which the Gramians were defined in a specified frequency band rather than using frequency weightings. However, this approach [11] suffered from the instability issue in some cases, even for an originally stable system. Additionally, GJ [11] didn't provide a *priori* error bound. Therefore, using Wang et al. [6], Gugercin and Antoulas (GA) [7] made changes to [11] and determined a stable ROM with a *priori* error bound. Numerous extensions of [11] are also developed to overcome its instability drawback using different Gramian structures such as [12], [13], [14], [15], [16], [17]. Using cross-Gramians, Jazlan et al. [18] developed the frequency-limited MOR for discrete-time bilinear systems, leading to a shorter algorithmic computational time. Subsequently, Zulfiqar et al. [15] created a revised incremental approach that needs a small adjustment to the eigenvalues of the intermediary symmetric matrices in order to produce reliable ROM. Jazlan et al. [19] also used frequency-limited MOR to streamline intricate FIR digital filters.

As observed in [15], the eigenvalues of intermediate matrices of some of the existing methods become significantly large. It results in larger approximation errors due to uneven effects on the eigenvalues while developing a new configuration. Therefore, we present a novel frequency-limited TBR (FLTBR) algorithm for linear time-invariant, continuous-time systems using a new configuration of Gramians. Using the new configuration, the novel controllability and observability Gramians are also shown, which satisfy the respective Lyapunov equations.

II. PROPOSED FLTBR TECHNIQUE

Consider the full-order stable model as

$$\mathcal{G}(s) = \mathcal{C}(sI - \mathcal{A})^{-1}\mathcal{B} + \mathcal{D}, \tag{1}$$

where $\mathcal{A} \in \mathbb{R}^{n\times n}, \mathcal{B} \in \mathbb{R}^{n\times p}, \mathcal{C} \in \mathbb{R}^{l\times n}, \mathcal{D} \in \mathbb{R}^{l\times p}$ is an n^{th} order minimal realization. The proposed objective is to determine the following k^{th} order stable and minimal realization

$$\mathcal{G}_k(s) = \mathcal{C}_k(sI - \mathcal{A}_k)^{-1}\mathcal{B}_k + \mathcal{D}_k, \tag{2}$$

such that the error $\|(\mathcal{G}(s) - \mathcal{G}_k(s)\|_\infty$ is minimized in a desired frequency band $[\omega_1, \omega_2]$ where $\mathcal{A}_k \in \mathbb{R}^{k\times k}, \mathcal{B}_k \in \mathbb{R}^{k\times p}, \mathcal{C}_k \in \mathbb{R}^{l\times k}, \mathcal{D}_k \in \mathbb{R}^{l\times p}$.

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The Gramians for $\mathcal{G}(s)$ in continuous-time domain can be represented as [11]

$$\mathcal{P}_g(\omega) = \frac{1}{2\pi}\int_{\delta\omega}(j\omega I - \mathcal{A})^{-1}\mathcal{B}\mathcal{B}^T(-j\omega I - \mathcal{A}^T)^{-1}d\omega \quad (3)$$

$$\mathcal{Q}_g(\omega) = \frac{1}{2\pi}\int_{\delta\omega}(-j\omega I - \mathcal{A}^T)^{-1}\mathcal{C}^T\mathcal{C}(j\omega I - \mathcal{A})^{-1}d\omega \quad (4)$$

where $\delta\omega$ is the the desired frequency band $[\omega_1, \omega_2]$. The Gramians $\mathcal{P}_{gj} = \mathcal{P}_g(\omega_2) - \mathcal{P}_g(\omega_1)$ and $\mathcal{Q}_{gj} = \mathcal{Q}_g(\omega_2) - \mathcal{Q}_g(\omega_1)$ satisfy the following Lyapunov-like equations

$$\mathcal{A}\mathcal{P}_{gj} + \mathcal{P}_{gj}\mathcal{A}^T + \mathcal{X}_{g1} = 0 \quad (5)$$

$$\mathcal{A}^T\mathcal{Q}_{g1} + \mathcal{Q}_{g1}\mathcal{A} + \mathcal{Y}_{g1} = 0 \quad (6)$$

where the matrices $\mathcal{X}_{g1}$ and $\mathcal{Y}_{g1}$ can be determined as:

$$\mathcal{X}_{g1}=(\mathcal{S}(\omega_2) - \mathcal{S}(\omega_1))\mathcal{B}\mathcal{B}^T + \mathcal{B}\mathcal{B}^T(\mathcal{S}^*(\omega_2) - \mathcal{S}^*(\omega_1)) \quad (7)$$

$$\mathcal{Y}_{g1}=(\mathcal{S}^*(\omega_2) - \mathcal{S}^*(\omega_1))\mathcal{C}^T\mathcal{C} + \mathcal{C}^T\mathcal{C}(\mathcal{S}(\omega_2) - \mathcal{S}(\omega_1)) \quad (8)$$

where

$$\mathcal{S}(\omega) = \frac{j}{2\pi}\ln\left[(j\omega I + \mathcal{A})(-j\omega I + \mathcal{A})^{-1}\right] \quad (9)$$

and $\mathcal{S}^*(\omega)$ is the conjugate transpose of $\mathcal{S}(\omega)$.

Now, the eigenvalue decomposition (EVD) of the symmetric matrices obtained in (7)-(8) gives

$$\mathcal{X}_{g1} = U_{kd}S_{kd}U_{kd}^T = U_{kd}\,\mathrm{diag}[S_{kd1}, S_{kd2}]U_{kd}^T \quad (10)$$

$$\mathcal{Y}_{g1} = V_{kd}H_{kd}V_{kd}^T = V_{kd}\,\mathrm{diag}[H_{kd1}, H_{kd2}]V_{kd}^T \quad (11)$$

where $S_{kd1} = diag\{s_1, s_2, \cdots, s_p\}$, $S_{kd2} = diag\{s_{p+1}, s_{p+2}, \cdots, s_n\}$, $H_{kd1} = diag\{h_1, h_2, \cdots, h_p\}$ and $H_{kd2} = diag\{h_{p+1}, h_{p+2}, \cdots, h_n\}$. Here, p is the number of positive eigenvalues and s_n and h_n are the smallest singular values at (n, n) entry of the S_{kd} and H_{kd} matrices while arranging the terms in descending order.

Using (10)-(11), the proposed fictive input and output matrices are formed as

$$\mathcal{B}_{kd} = \begin{cases} U_{kd}\begin{bmatrix} S_{kd1}^{\frac{1}{2}} & 0 \\ 0 & \left|\frac{S_{kd2}}{s_n}\right|^{\frac{1}{2}}\end{bmatrix}, & \text{for } s_n < 0 \\ U_{kd}S_{kd}^{1/2}, & \text{for } s_n \geq 0 \end{cases}, \quad (12)$$

$$\mathcal{C}_{kd} = \begin{cases} \begin{bmatrix} H_{kd1}^{\frac{1}{2}} & 0 \\ 0 & \left|\frac{H_{kd2}}{h_n}\right|^{\frac{1}{2}}\end{bmatrix} V_{kd}^T, & \text{for } h_n < 0 \\ H_{kd1}^{1/2}V_{kd}^T, & \text{for } h_n \geq 0 \end{cases}, \quad (13)$$

which generate the proposed Gramian matrices P_{kd} and Q_{kd} and fulfill the following Lyapunov equations

$$\mathcal{A}\mathcal{P}_{kd} + \mathcal{P}_{kd}A_a^T + \mathcal{B}_{kd}\mathcal{B}_{kd}^T = 0, \quad (14)$$

$$\mathcal{A}^TQ_{kd} + \mathcal{Q}_{kd}\mathcal{A} + \mathcal{C}_{kd}^T\mathcal{C}_{kd} = 0, \quad (15)$$

Now, the singular value decomposition (SVD) of the proposed Gramians is performed as follows

$$\mathcal{P}_{kd} = U_kS_kH_k^T \quad (16)$$

$$\mathcal{Q}_{kd} = V_kH_kH_k^T \quad (17)$$

Using (16)-(17), the transformation matrix [1] for the proposed FLTBR is determined as

$$\mathcal{T}_{kd} = U_kS_k^{\frac{1}{2}}V_kH_k^{\frac{-1}{2}} \quad (18)$$

which satisfy

$$\mathcal{T}_{kd}^T\mathcal{Q}_{kd}\mathcal{T}_{kd} = \mathcal{T}_{kd}^{-1}\mathcal{P}_{kd}\mathcal{T}_{kd}^{-T} = \sum, \quad (19)$$

where $\sum = diag(\epsilon_1, \epsilon_2, \cdots, \epsilon_n)$, $\epsilon_p \geq \epsilon_{p-1}, p = 1, 2, \cdots, n-1$, $\epsilon_k > \epsilon_{k+1}$. By performing the above transformation, the proposed FLTBR model is obtained.

A. Algorithm

The proposed FLTBR algorithm is summarized for the frequency interval $[\omega_1, \omega_2]$ as follows

1) Find the matrices $\mathcal{X}_{g1}$ and $\mathcal{Y}_{g1}$ using (7)-(8) for the given frequency interval.
2) Perform EVD of $\mathcal{X}_{g1}$ and $\mathcal{Y}_{g1}$ as shown in (10)-(11) and construct the matrices S_{kd2} and H_{kd2}.
3) Now, find the new fictive matrices B_{kd} and C_{kd} using (12)- (13) to evaluate the proposed Gramians $\mathcal{P}_{kd}$ and $\mathcal{Q}_{kd}$ as given in (14)-(15).
4) Perform the SVD of Gramians $\mathcal{P}_{kd}$ and $\mathcal{Q}_{kd}$ as shown in (16)-(17).
5) Obtain matrix $\mathcal{T}_{kd}$ [1] as shown in (19) to transform the given $\mathcal{G}(s)$ as

$$\mathcal{A}_k = \mathcal{T}_{kd}^{-1}\mathcal{A}\mathcal{T}_{kd} = \begin{bmatrix} A_{k11} & A_{k12} \\ A_{k21} & A_{k22}\end{bmatrix}, \quad (20)$$

$$\mathcal{B}_k = \mathcal{T}_{kd}^{-1}\mathcal{B} = \begin{bmatrix} \mathcal{B}_{k1} & \mathcal{B}_{k2}\end{bmatrix}^T, \quad (21)$$

$$\mathcal{C}_k = \mathcal{C}\mathcal{T}_{kd} = \begin{bmatrix} \mathcal{C}_{k1} & \mathcal{C}_{k2}\end{bmatrix}. \quad (22)$$

6) Finally, the above transformation yields the required ROM as $\mathcal{G}_k = \mathcal{C}_{k1}\left(sI - \mathcal{A}_{k11}\right)^{-1}\mathcal{B}_{k1} + D$ where $A_{a11} \in \mathbb{R}^{k\times k}$.

Remark 1: The positive semi-definiteness of $\mathcal{X}_{g1}(\Omega)$ and $\mathcal{Y}_{g1}(\Omega)$ isn't guaranteed, which leads to instability of [11]. On the other hand, the suggested method yields stable ROMs as $\mathcal{B}_{kd}\mathcal{B}_{kd}^T \geq 0, \mathcal{C}_{kd}^T\mathcal{C}_{kd} \geq 0, \mathcal{X}_{g1}(\Omega) \leq \mathcal{B}_{kd}\mathcal{B}_{kd}^T, \mathcal{Y}_{g1}(\Omega) \leq \mathcal{C}_{kd}^T\mathcal{C}_{kd}, \mathcal{P}_{kd}(\Omega) > 0, \mathcal{Q}_{kd}(\Omega) > 0$.

III. SIMULATION OUTCOMES

Example 1

Consider the following 8th-order Chebyshev type-1 analog band-pass filter with 20dB peak-to-peak ripple in the pass-band:

$$\mathcal{G}(s) = \frac{\begin{array}{c}482.9s^4 + 3.162\times10^{-14}s^3 + 8.399\times10^{-13}s^2 \\ + 1.707\times10^{-23}s - 5.719\times10^{-27}\end{array}}{\begin{array}{c}s^8 + 0.9178s^7 + 256.4s^6 + 157.7s^5 + \\ 157.7s^4 + 2366s^3 + 5.77e04s^2 + 3098s + 5.063e04\end{array}}.$$

First, the ROMs for the aforementioned filter are obtained using existing [11], [7], [10] and proposed methodologies, taking into account the frequency intervals $[2.5, 4.5]$ and $[3, 5]$ rad/sec. Table I indicates the location of the ROM

TABLE I

POLE LOCATIONS OF ROMS IN [2.5,3.5] RAD/SEC FOR EXAMPLE 1

Original	Order	GJ [11]	Proposed
$-0.125 \pm 14.01i$	1	**4.156e-8**	-0.0316
$-0.255 \pm 7.387i$	2	-1.9346, -7.7506	$-0.0763 \pm 2.346i$
$-0.069 \pm 2.028i$	3	**7.7506**, 0.00 -1.9346	$-0.0894 \pm 2.393i$ $-0.0203 + 0.000i$
$-0.009 \pm 1.071i$	4	$-0.1862 \pm 7.104i$, $-0.0553 \pm 2.110i$	$-0.2367 \pm 8.475i$ $-0.0657 \pm 1.982i$

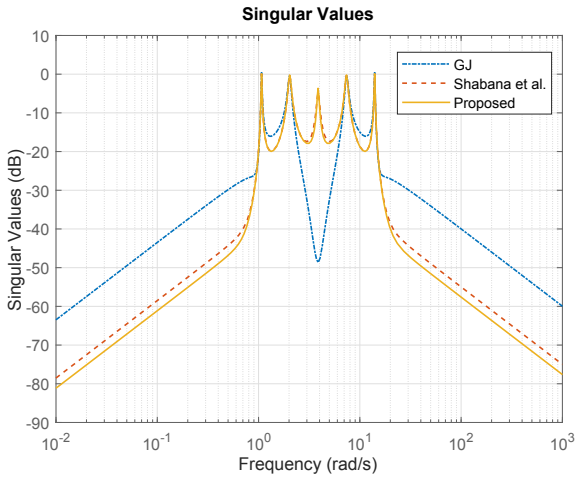


Fig. 1. Frequency response error plots of 3rd order ROMs for Example 1 with interval $[3, 5]$ rad/sec

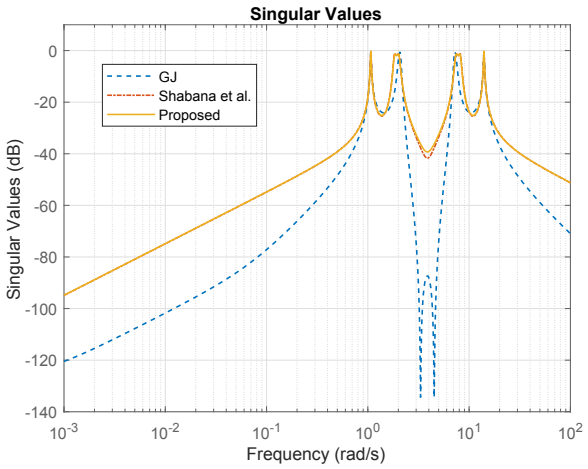


Fig. 2. Frequency response error plots of 4th order ROMs for Example 1 with interval $[3, 5]$ rad/sec

poles for $[3, 5]$ rad/ sec. It is evident from the data that the first and third-order models via the GJ technique [11] are unstable, with a pole located in the right half of the s-plane. Conversely, since all poles in every case lie in the left half of the s-plane, the suggested method yields stable ROMs. The error $\|\mathcal{G}(s) - \mathcal{G}_k(s)\|_\infty$ of the ROMs derived by GJ [11], GA [7], Shabana et al. [10] and the suggested

TABLE II

ERROR OF ROMS IN THE INTERVAL [3,5] RAD/SEC FOR EXAMPLE 1

Order	GJ [11]	GA [7]	Shabana et al. [10]	Proposed
1	1.0004	1.0700	1.3367	1.0004
2	1.0546	1.0148	1.0142	1.0107
3	1.0546	1.0148	1.0142	1.0107
4	0.9988	0.9727	0.9730	0.9728

TABLE III

ERROR OF ROMS IN THE INTERVAL [2.5,4.5] RAD/SEC FOR EXAMPLE 1

Order	GJ [11]	GA [7]	Shabana et al. [10]	Proposed
1	1.0825	1.0761	1.2747	1.0146
2	1.3721	1.0229	1.0238	1.0175
3	1.1004	1.0108	1.0106	1.0086
4	1.1172	0.9896	0.9896	0.9893

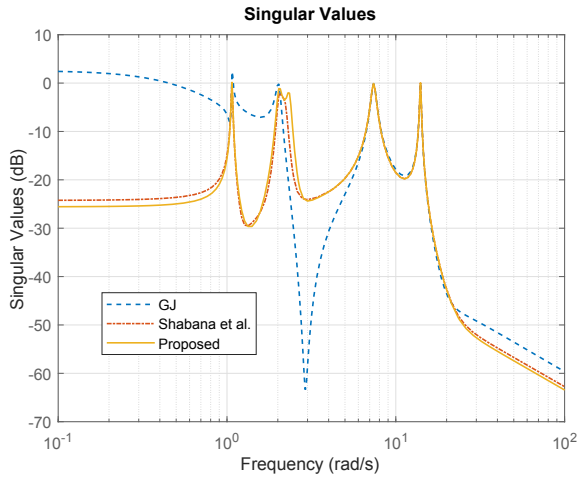


Fig. 3. Frequency response error plots of 2nd order ROMs for Example 1 with interval $[2.5, 4.5]$ rad/sec

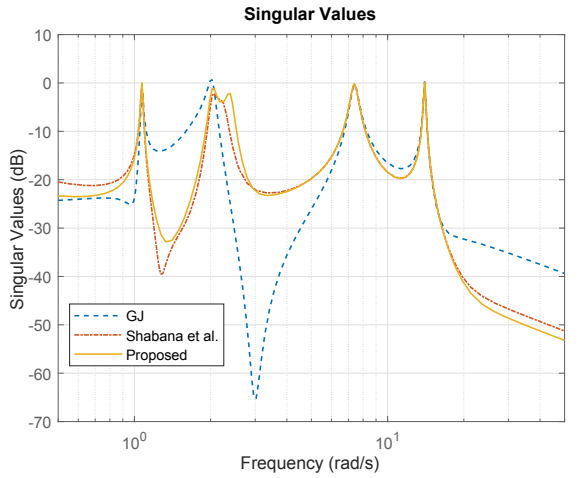


Fig. 4. Frequency response error plots of 3rd order ROMs for Example 1 with interval $[2.5, 4.5]$ rad/sec

method for frequency intervals $[3, 5]$ rad/sec and $[2.5, 4.5]$ rad/sec, respectively, are displayed in Tables II and III. The

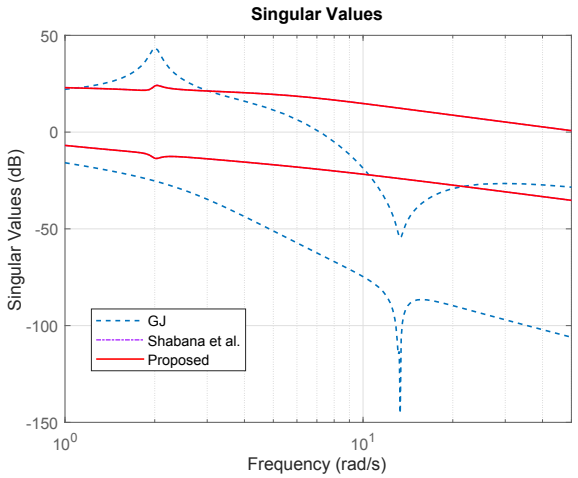


Fig. 5. Frequency response error plots of 3rd order ROMs for Example 2 with interval $[12, 15]$ rad/sec

suggested strategy offers a superior error approximation, as the tables illustrate. Furthermore, Figs. (1)-(2) and (3)-(4) exhibit the singular value plots for the modeling errors of the ROMs by GJ [11], Shabana et al. [10] and the suggested methodologies for the frequency intervals $[3, 5]$ rad/sec and $[2.5, 4.5]$ rad/sec, respectively. As the graphs demonstrate, a major shift in the eigenvalues causes a significant error for existing methods. The figures make it evident that the presented algorithm outperforms the current methods as per the results.

Example 2

Consider a 6th order strictly proper multi-input multi-output (MIMO) system [20] as

$$\mathcal{A} = \begin{bmatrix} -13 & 1 & 0 & 0 & 0 & 0 \\ -75 & 0 & 1 & 0 & 0 & 0 \\ -210 & 0 & 0 & 1 & 0 & 0 \\ -320 & 0 & 0 & 0 & 1 & 0 \\ -600 & 0 & 0 & 0 & 0 & 1 \\ -80 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 0 & 1 \\ 1 & 0.6 \\ 2 & 0.9 \\ 5 & 0.3 \\ 1 & 0.6 \\ 0.5 & 1 \end{bmatrix},$$

$$\mathcal{C} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 \end{bmatrix}.$$

The ROMs using the proposed and existing techniques [11], [10] are determined for frequency interval [12, 15] rad/sec. Table IV indicates that the first and fourth-order ROMs by GJ [11] are unstable while proposed ROMs for all orders are stable. Fig. 5 displays the singular value error plots for the third order ROMs. Additionally, Table V provides the error comparison of the ROMs. Like the last example, the GJ [11] method generates less error in the intended frequency interval but yields unstable models for a few cases as shown in Table IV. The responses for the specified frequency intervals provided in Fig. 5 show that the ROMs created using the recommended technique and [10] are similar. Therefore, considering the results, it can be said that the suggested approach provides satisfactory and comparable results due minimization of error in the specified interval.

TABLE IV

POLE LOCATIONS OF ROMS FOR EXAMPLE 2

Original	Order	GJ [11]	Proposed
-0.1433	1	**0.8129**	-0.0485
$-4.0326 \pm 3.68i$	2	$-5.1023 \pm 5.714i$	$-0.0786 \pm 2.045i$
$-0.073 \pm 2.006i$	3	$-4.7803 \pm 4.774i$ -2.6625	$-0.1042 \pm 2.020i$ $-1.7712 + 0.000i$
-4.6458	4	$-4.5737 \pm 3.557i$, **0.6404**, -2.4401	$-0.0742 \pm 2.009i$ $-1.4698 \pm 4.223i$

TABLE V

ERROR OF ROMS IN THE INTERVAL [12,15] RAD/SEC FOR EXAMPLE 2

Order	GJ [11]	GA [7]	Shabana et al. [10]	Proposed
1	164.5429	159.2262	163.3265	161.0858
2	156.5207	24.5093	25.1242	24.7867
3	150.3773	21.2173	21.5797	21.4637
4	143.704	9.5236	9.607	9.5781

IV. CONCLUSIONS

This study provides a new truncated balanced realization approach that uses new Gramian structures to minimize the error in designated frequency ranges. Numerical example of an eighth-order elliptic filter and a 6th order MIMO system are used to validate the recommended strategy, which produces a better error in the targeted frequency regions than the prevalent methods. Moreover, it is demonstrated that the suggested approach produces stable ROMs.

REFERENCES

[1] B. Moore, "Principal component analysis in linear systems: Controllability, observability, and model reduction," *IEEE transactions on automatic control*, vol. 26, no. 1, pp. 17–32, 1981.

[2] K. Glover, "All optimal hankel-norm approximations of linear multivariable systems and their $\mathcal{L}_\infty$-error bounds," *International journal of control*, vol. 39, no. 6, pp. 1115–1193, 1984.

[3] Y. Liu and B. D. Anderson, "Singular perturbation approximation of balanced systems," *International journal of control*, vol. 50, no. 4, pp. 1379–1405, 1989.

[4] D. F. Enns, "Model reduction with balanced realizations: An error bound and a frequency weighted generalization," in *Proc. 23rd IEEE conference on decision and control*, 1984, pp. 127–132.

[5] V. Sreeram, B. Anderson, and A. Madievski, "New results on frequency weighted balanced reduction technique," in *Proc. 1995 American Control Conference-ACC'95*, vol. 6, 1995, pp. 4004–4009.

[6] G. Wang, V. Sreeram, and W. Liu, "A new frequency-weighted balanced truncation method and an error bound," *IEEE Transactions on Automatic Control*, vol. 44, no. 9, pp. 1734–1737, 1999.

[7] S. Gugercin and A. C. Antoulas, "A survey of model reduction by balanced truncation and some new results," *International Journal of Control*, vol. 77, no. 8, pp. 748–766, 2004.

[8] V. Sreeram and S. Sahlan, "Improved results on frequency-weighted balanced truncation and error bounds," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 11, pp. 1195–1211, 2012.

[9] D. Kumar and V. Sreeram, "Factorization-based frequency-weighted optimal hankel-norm model reduction," *Asian Journal of Control*, vol. 22, no. 5, pp. 2106–2118, 2020.

[10] S. Bashrat, M. Imran, S. Akram, A. Wakeel, N. Anwar Baig, and A. Zaheer Ud-Din, "Improved model order reduction techniques with error bounds," *International Journal of Systems Science*, vol. 55, no. 4, pp. 687–700, 2024.

[11] W. Gawronski and J.-N. Juang, "Model reduction in limited time and frequency intervals," *International Journal of Systems Science*, vol. 21, no. 2, pp. 349–376, 1990.
[12] A. Ghafoor and V. Sreeram, "Model reduction via limited frequency interval Gramians," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 55, no. 9, pp. 2806–2812, 2008.
[13] M. Imran and A. Ghafoor, "Stability preserving model reduction technique and error bounds using frequency-limited Gramians for discrete-time systems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 61, no. 9, pp. 716–720, 2014.
[14] ——, "Stability preserving model reduction technique for 1-d and 2-d systems with error bounds," *IEEE transactions on circuits and systems II: Express briefs*, vol. 69, no. 3, pp. 1084–1088, 2021.
[15] U. Zulfiqar, M. Imran, A. Ghafoor, and M. Liaquat, "A new frequency-limited interval gramians-based model order reduction technique," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 64, no. 6, pp. 680–684, 2016.
[16] V. Sharma and D. Kumar, "Frequency-weighted and frequency interval gramian framework-based model reduction using singular value decomposition," *IMA Journal of Mathematical Control and Information*, vol. 41, no. 1, pp. 57–72, 2024.
[17] M. Latif, H. Ambreen, F. Hassan, and M. Imran, "Computationally efficient reduced order modeling of dfig-based wind turbines: A novel frequency-weighted and limited model reduction approach with error bounds," *IEEE Access*, vol. 12, pp. 54 299–54 315, 2024.
[18] A. Jazlan, V. Sreeram, H. R. Shaker, and R. Togneri, "Frequency interval balanced truncation of discrete-time bilinear systems," *Cogent Engineering*, vol. 3, no. 1, p. 1203082, 2016.
[19] A. Jazlan, U. Zulfiqar, V. Sreeram, D. Kumar, R. Togneri, and H. F. M. Zaki, "Frequency interval model reduction of complex fir digital filters," *Numerical Algebra, Control and Optimization*, vol. 9, no. 3, pp. 319–326, 2019.
[20] V. Sharma and D. Kumar, "Confined frequency-interval gramian framework-based balanced model reduction," *IETE Journal of Research*, vol. 70, no. 7, pp. 6310–6317, 2024.