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# Robust $\mathscr{H}_\infty$ Performance of Linear Interconnected Markovian Jump Systems with Time-varying Delays via Sampled-Data Control

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***Abstract*— This paper investigates the stochastic stability and stabilization issues of a linear interconnected Markovian jump system (LIMJS) with time-varying delay in the interconnected term. To address this, a mode-dependent sampled-data control is designed, and a new looped-Lyapunov functional (LLF) is constructed, utilizing the information of the sampling instants and time-delay. The linear matrix inequality based, delay-dependent sufficient conditions (DDSCs) are derived with the help of the new LLF, which ensures the stochastic stability of LIMJS with $\mathscr{H}_\infty$ performance level. Finally, a numerical example and its simulation results are given to illustrate the effectiveness of the proposed DDSCs.**

***Index Terms*— Interconnected system, Markovian jump system, Sampled-data control, looped-Lyapunov functional, $\mathscr{H}_\infty$ performance.**

## I. INTRODUCTION

In recent decades, interconnected systems have attracted substantial attention owing to their significant developments in practical applications and theoretical development. Several practical systems, such as multi-vehicle systems [1] and power systems [2], can be described as interconnected systems. As we know that an interconnected system comprises several subsystems that interact with each other. Therefore, the investigation of the qualitative analysis can lead to an understanding of the stability and stabilization issues under suitable control schemes in complex environments.

On the other hand, Markovian jump systems (MJSs) are a distinct type of hybrid system [3] capable of modeling many practical systems, such as aerospace systems [4], robotics [5], and power systems [6]. A wide range of practical systems may experience major structural or parametric changes due to unpredictable environmental disturbances, device errors, or repairs. These changes often cause sudden shifts in the system's state, which are not historically dependent, indicating that the transitions usually follow a Markov process, leading to the formation of a MJS.

Moreover, control schemes are essential for analyzing the stability and stabilization issues of a system. They provide the necessary framework to ensure that the system behaves predictably under varying conditions. Additionally, effective control schemes help mitigate potential disruptions, thereby enhancing the overall reliability and performance of the system. Recently, the sampled-data control (SDC) approach has garnered significant importance as a solution to the stabilization issue of control systems, primarily due to its implementation and control activities during the sampling period. On the other hand, time-dependent Lyapunov functionals (LFs) have been proposed recently and utilized to obtain less conservative results [7]. Some of the related works to SDC are: SDC design for interconnected power systems [8], stability analysis of interconnected systems [9], stability and stabilization for Takagi-Sugeno (T-S) fuzzy large scale inteconnected power systems [10] and state estimation problem for T-S fuzzy MJS [11].

In literature, a distributed finite-time stochastic control was designed in [12] to investigate the finite-time stochastic stability (SS) of spatially interconnected MJS (IMJS). In [13], the authors employed observer-based tracking control to examine the finite-time stability of a saturated IMJS. In [14], the author designed a sliding mode control to analyze the SS of the LIMJS. Several existing studies are related to IMJS, exploring various control strategies such as load frequency control [15], decentralized adaptive sliding mode control [16], and finite-time $\mathscr{H}_\infty$ filtering design [17].

From the above literature, we observe that the stability and stabilization issues of linear IMJS (LIMJS) have been thoroughly investigated under various control strategies. However, SDC has been utilized in very few studies to examine these problems. This gap in the literature motivated the study of SS and stabilization issues of LIMJS with time-varying delay in the interconnected term under mode-dependent SDC (MDSDC) based on a new looped-Lyapunov functional (LLF). The primary goals of this article are detailed below.

- This article explores the SS of LIMJS with time-varying delay in the interconnected term through a MDSDC.
- A novel LLF is constructed employing detailed information of the time-delay and sampling instants. The DDSCs are derived to ensure the SS of LIMJS with $\mathscr{H}_\infty$ performance level.
- At last, the efficacy of the proposed method is validated through a numerical example.

*Notations*: In this article, $\mathbb{R}^{n_i}$, $\mathbb{R}^{n_i \times m_i}$ and $\mathbb{N}$ represents the $n$-dimensional Euclidean space, the set of all $n_i \times m_i$ real matrices and the set of all natural numbers respectively. The positive and negative definite symmetric matrix are represent by $X > 0$ and $X < 0$ respectively. $I$ and $0$ are identity and zero matrices with suitable dimensions respectively, the symbol " * " represent the symmetric terms in a matrix. $diag\{...\}$ represents the block diagonal matrix. $\mathcal{L}_2[0, \mathcal{T}]$ will be used for vector-valued functions. $i.e.$, it satisfies the condition $\int_0^{\mathcal{T}} w_i^T(t) w_i(t) dt < \infty$ where $\mathcal{T} \in (0, \infty)$ is given

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finite time. $(\Omega, \mathscr{F}, P)$ represent a standard probability space where $\Omega$, $\mathscr{F}$ and $P$ represent the sample space, $\sigma$-algebra of subsets of $\Omega$ and probability measure respectively. $\mathbb{E}\{\cdot\}$ represent the mathematical expectation operator. $He\{A\}$ represents $A + A^T$.

## II. Problem Formulation and Preliminaries

Let us consider a LIMJSs containing $N \in \mathbb{N}$ subsystems then the $i^{th}$ subsystem with interconnection delay is described by

$$\begin{aligned}\dot{x}_i(t) &= A_i(r(t))x_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij}(r(t))x_j(t-\tau_{ij}(t)) \\ &\quad + B_i(r(t))u_i(t) + D_i(r(t))w_i(t) \\ y_i(t) &= C_i(r(t))x_i(t) \end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$, $y_i(t) \in \mathbb{R}^{n_i}$ and $w_i(t) \in \mathbb{R}^{q_i}$ is represent the state vector, control input, measured output and disturbance input of $i^{th}$ subsystem respectively. $w_i(t)$ belongs to $\mathcal{L}_2[0,\infty)$; $\tau_{ij}(t)$, $i, j \in \mathcal{N}$ represent the time-varying delay and satisfying

$$0 \leq \tau_{ij}(t) \leq h_{ij}, \qquad \dot{\tau}_{ij}(t) \leq \tau_{ij} < 1 \tag{2}$$

where $h_{ij}$ and $\tau_{ij}$ are known constants. The matrices $A_i(r(t)) \in \mathbb{R}^{n_i\times n_i}, A_{ij}(r(t)) \in \mathbb{R}^{n_i\times n_i}, B_i(r(t)) \in \mathbb{R}^{n_i\times m_i}, D_i(r(t)) \in \mathbb{R}^{n_i\times q_i}$ and $C_i(r(t)) \in \mathbb{R}^{m_i\times n_i}$ are known real matrices of the continuous-time Markov process $\{r(t), t \geq 0\}$ takes values in the finite discrete set $\mathcal{S} = \{1, 2, ..., s\}, s \in \mathbb{N}$ and has the transition probabilities

$$Pr\{r(t+\Delta) = \beta | r(t) = \alpha\} = \begin{cases} \pi_{\alpha\beta}\Delta + o(\Delta), & if \alpha \neq \beta \\ 1 + \pi_{\alpha\beta}\Delta + o(\Delta), & if \alpha = \beta \end{cases} \tag{3}$$

where $\Delta > 0$, $\lim\limits_{\Delta\to 0^+} \frac{o(\Delta)}{\Delta} = 0$ and $\pi_{\alpha\beta}$ represent the transition rate (TR) from mode $\alpha$ to $\beta$ at time $t$ and $t+\Delta$ respectively satisfying $\pi_{\alpha\beta} \geq 0$, for all $\alpha \neq \beta$ and $\pi_{\alpha\alpha} = -\sum_{\substack{\beta=1\\ \alpha\neq\beta}}^{s} \pi_{\alpha\beta}$.

For each possibe value of $r(t) = \alpha$, $\alpha \in \mathcal{S}$, we denote the coefficient matrices $A_i^\alpha$, $A_{ij}^\alpha$, $B_i^\alpha$, $D_i^\alpha$ and $C_i^\alpha$ for clarity. From the zero-order hold function with a set of holding times $0 = t_0 < t_1 < ... < t_k < t_{k+1} < ..., t_k \in \mathbb{R}^+$, $\lim\limits_{k\to\infty} t_k = +\infty$ and we define $0 \leq \underline{q} \leq t_{k+1} - t_k = h_k \leq \bar{q}$ where $\underline{q}$ and $\bar{q}$ are the lower bound and upper bound of $h_k$ respectively.

We have the following MD SDC for the LIMJSs (1),

$$u_i(t) = K_i^\alpha x_i(t_k), \quad t_k \leq t < t_{k+1}, \ \ k \in \mathbb{N} \tag{4}$$

where $K_i^\alpha$ is the MD control gain matrix for $i^{th}$ subsystems.

Hence, we get the corresponding closed-loop LIMJS (1) as follows:

$$\begin{aligned}\dot{x}_i(t) &= A_i^\alpha x_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij}^\alpha x_j(t-\tau_{ij}(t)) \\ &\quad + B_i^\alpha K_i^\alpha x_i(t_k) + D_i^\alpha w_i(t) \\ y_i(t) &= C_i^\alpha x_i(t). \end{aligned} \tag{5}$$

*Definition 1:* [18] The infinitesimal operator performing on a function $\mathcal{V}(x(t), r(t), t)$ is given by

$$\mathcal{V}(x(t), r(t), t) = \lim_{\Delta\to 0^+} \frac{1}{\Delta}\Big\{\mathbb{E}\big[\mathcal{V}\big(x(t+\Delta), r(t+\Delta), t + \Delta|_{(x(t),r(t),t)}\big)\big] - \mathcal{V}(x(t), r(t), t)\Big\}.$$

*Definition 2:* [19] The LIMJS (1) is stochastically stable with $\mathscr{H}_\infty$ performance attenuation level $\gamma$ if the below inequalities hold:

(i) The closed-loop LIMJS (5) with $w_i(t) \equiv 0$ is stochastically stable. *i.e.*,

$$\mathbb{E}\Big\{\int_0^\infty ||x_i(t)||^2 dt|_{(x_i(t_0),r_0)}\Big\} < \infty \tag{6}$$

for all $(x_i(t_0), r_0)$ where $x_i(t_0)$ and $r_0 \in \mathcal{S}$ represent the initial state and initial mode respectively.

(ii) Under zero initial condition, $y_i(t)$ satisfies

$$||y_i(t)||_2 < \gamma ||w_i(t)||_2 \tag{7}$$

for every non-zero $w_i(t) \in \mathcal{L}_2[0,\infty)$ where $\gamma > 0$ is a specified level of disturbance attenuation.

## III. Main Result

In this section, a novel time-dependent looped LF is constructed and the MDSDC is sketched to minimize the $\mathscr{H}_\infty$ performance of the LIMJSs. For simplicity, the following are defined:
$\Phi_1(t) = t - t_k$, $\Phi_2(t) = t_{k+1} - t$, $\Phi_{3i}(t) = x_i(t) - x_i(t_k)$ and $\zeta_i^T(t) = [x_i^T(t) \ \ x_i^T(t_k)]$.

*Theorem 1:* For given MD control gain matrices and any scalars $k, 0 < \gamma, 0 < \underline{q} \leq \bar{q}$ and if there exist real matrices $\mathscr{P}_i^\alpha > 0, (\alpha \in \mathcal{S}), \mathscr{Q}_i > 0, \mathscr{R}_i > 0, \mathscr{T}_i > 0$, $\mathscr{M}_{ij} > 0, \mathbb{X}_i > 0$ and $\mathbb{L}_{1i}, \mathbb{L}_{2i}$ are any matrices with appropriate dimensions, $i, j \in \mathcal{N}$ such that for all $h_k \in [\underline{q}, \bar{q}]$

$$\Lambda_1 = \begin{bmatrix} \bar{\Xi}_{ij} + h_k\Delta_{ij}^1 & \frac{\bar{q}}{\sqrt{8}}\Sigma_i \\ * & -He\{\mathbb{X}_i\} \end{bmatrix} < 0 \tag{8}$$

$$\Lambda_2 = \begin{bmatrix} \bar{\Xi}_{ij} + h_k\Delta_{ij}^2 & \frac{\bar{q}}{\sqrt{8}}\Sigma_i \\ * & -He\{\mathbb{X}_i\} \end{bmatrix} < 0 \tag{9}$$

where
$\bar{\Xi}_{11} = \sum_{\beta=1}^{s} \pi_{\alpha\beta}\mathscr{P}_i^\beta + \mathscr{Q}_i - \mathscr{T}_i + \mathscr{M}_{ji} + He\{\mathbb{L}_{1i}^T A_i^\alpha\} + C_i^{\alpha T} C_i^\alpha$,
$\bar{\Xi}_{12} = \mathscr{T}_i + \mathbb{L}_{1i}^T B_i^\alpha K_i^\alpha$,
$\bar{\Xi}_{13} = \mathscr{P}_i^\alpha + kA_i^{T\alpha}\mathbb{L}_{1i} - \mathbb{L}_{1i}^T$, $\bar{\Xi}_{15} = \mathbb{L}_{1i}^T A_{ij}^\alpha$,
$\bar{\Xi}_{17} = \mathbb{L}_{1i}^T D_i^\alpha$, $\bar{\Xi}_{23} = kK_i^{T\alpha} B_i^{T\alpha}\mathbb{L}_{1i}$,
$\bar{\Xi}_{33} = He\{-k\mathbb{L}_{1i}^T\} + \frac{\bar{q}^2}{8}He\{\mathbb{X}_i\}$, $\bar{\Xi}_{35} = k\mathbb{L}_{1i}^T A_{ij}^\alpha$,
$\bar{\Xi}_{37} = k\mathbb{L}_{1i}^T D_i^\alpha$, $\bar{\Xi}_{44} = -(1-\tau_{ij})\mathscr{Q}_i$, $\bar{\Xi}_{55} = -(1-\tau_{ij})\mathscr{M}_{ij}$,
$\Delta_{11}^1 = -r_{11i}$, $\Delta_{12}^1 = -r_{12i}$, $\Delta_{22}^1 = -r_{22i}$, $\Delta_{11}^2 = r_{11i}$, $\Delta_{12}^2 = r_{12i}$, $\Delta_{13}^2 = \mathscr{T}_i$, $\Delta_{22}^2 = r_{22i}$, $\Delta_{23}^2 = -\mathscr{T}_i^T$,
$\mathscr{R}_i = \begin{bmatrix} r_{11i} & r_{12i} \\ * & r_{22i} \end{bmatrix}$, $\Sigma_i = \begin{bmatrix} r_{11i} & r_{12i} & 0 & 0 & 0 \end{bmatrix}^T$ then, the LIMJS (1) is stochastically stable with $\mathscr{H}_\infty$ performance level $\gamma$.

*Proof:* Let $\mathcal{V}_i(t) = \mathcal{V}_i(x_i(t), r(t), t)$. For every $r(t) \in \mathcal{S}$, consider a LF,

$$\mathcal{V}_i(t) = \sum_{n=1}^{4} \mathbb{V}_{ni}(t). \tag{10}$$

where

$$\begin{aligned}\mathbb{V}_{1i}(t) = \sum_{i=1}^{N} \big[ & x_i^T(t)\mathscr{P}_i(r(t))x_i(t) \\ & + \int_{t-\tau_{ij}(t)}^{t} x_i^T(s)\mathscr{Q}_i x_i(s)ds\big],\end{aligned} \tag{11}$$

$$\mathbb{V}_{2i}(t) = \sum_{i=1}^{N} \Phi_2(t)\Phi_1(t)\zeta_i^T(t)\mathscr{R}_i\zeta_i(t), \tag{12}$$

$$\mathbb{V}_{3i}(t) = \sum_{i=1}^{N} \Phi_2(t)\Phi_{3i}^T(t)\mathscr{T}_i\Phi_{3i}(t), \tag{13}$$

$$\mathbb{V}_{4i}(t) = \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} \int_{t-\tau_{ij}(t)}^{t} x_j^T(s)\mathscr{M}_{ij}x_j(s)ds \tag{14}$$

Taking infinitesimal operator of $\mathcal{V}_i(t)$, we arrive

$$\mathcal{V}_i(t) = \mathbb{V}_{1i}(t) + \mathbb{V}_{2i}(t) + \mathbb{V}_{3i}(t) + \mathbb{V}_{4i}(t). \tag{15}$$

$$\begin{aligned}\mathbb{V}_{1i}(t) = \sum_{i=1}^{N} \big[ & 2x_i^T(t)\mathscr{P}_i^{\alpha}\dot{x}_i(t) + \\ & x_i^T(t)\sum_{\beta=1}^{s}\pi_{\alpha\beta}\mathscr{P}_i^{\beta}x_i(t) + x_i^T(t)\mathscr{Q}_i x_i(t) \\ & - (1-\dot{\tau}_{ij}(t))x_i^T(t-\tau_{ij}(t))\mathscr{Q}_i x_i(t-\tau_{ij}(t))\big],\end{aligned} \tag{16}$$

$$\begin{aligned}\mathbb{V}_{2i}(t) = \sum_{i=1}^{N} \big[ & 2\Phi_2(t)\Phi_1(t)\Gamma_i^T(t)\Sigma_i\dot{x}_i(t) \\ & + \left[\Phi_2(t) - \Phi_1(t)\right]\zeta_i^T(t)\mathscr{R}_i\zeta_i(t)\big],\end{aligned} \tag{17}$$

$$\begin{aligned}\mathbb{V}_{3i}(t) = \sum_{i=1}^{N} \big[ & -\big(x_i^T(t)\mathscr{T}_i x_i(t) - 2x_i^T(t)\mathscr{T}_i x_i(t_k) \\ & + x_i^T(t_k)\mathscr{T}_i x_i(t_k)\big) + \Phi_2(t)\big(2x_i^T(t)\mathscr{T}_i\dot{x}_i(t) \\ & - 2\dot{x}_i^T(t)\mathscr{T}_i x_i(t_k)\big)\big],\end{aligned} \tag{18}$$

$$\begin{aligned}\mathbb{V}_{4i}(t) = \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} \big[ & x_j^T(t)\mathscr{M}_{ij}x_j(t) - (1-\dot{\tau}_{ij}(t)) \\ & (x_j^T(t-\tau_{ij}(t))\mathscr{M}_{ij}x_j(t-\tau_{ij}(t)))\big]\end{aligned} \tag{19}$$

where $\Gamma_i^T(t) = \big[x_i^T(t) \;\; x_i^T(t_k) \;\; \dot{x}_i^T(t) \;\; x_i^T(t-\tau_{ij}(t)) \;\; x_j^T(t-\tau_{ij}(t)) \;\; w_i^T(t)\big]$.
From $\mathbb{V}_{2i}$ and any positive definite $n\times n$ symmetric matrix $\mathbb{X}_i$, we get

$$\begin{aligned}& 2h_k\Phi_1(t)\Gamma_i^T(t)\Sigma_i\dot{x}_i(t) \\ & \quad \leq \frac{\bar{q}^2}{4}(\Gamma_i^T(t)\Sigma_i\mathbb{X}_i^{-1}\Sigma_i^T\Gamma_i(t) + \dot{x}_i^T(t)\mathbb{X}_i\dot{x}_i(t)).\end{aligned} \tag{20}$$

We know that,

$$\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} x_j^T(t)\mathscr{M}_{ij}x_j(t) = \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} x_i^T(t)\mathscr{M}_{ji}x_i(t). \tag{21}$$

For any non-singular matrices $\mathbb{L}_{1i}, \mathbb{L}_{2i}$ and from (1), we obtain

$$\begin{aligned}2\Big[x_i^T(t)\mathbb{L}_{1i}^T + \dot{x}_i^T(t)\mathbb{L}_{2i}^T\Big]\Big[ & -\dot{x}_i(t) + A_i^{\alpha}x_i(t) \\ & + \sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij}^{\alpha}x_j(t-\tau_{ij}(t)) + B_i^{\alpha}K_i^{\alpha}x_i(t_k)\Big] = 0.\end{aligned} \tag{22}$$

Substituting all the above (16)-(22) in (15), it becomes

$$\begin{aligned}\mathcal{V}_i(t) \leq \sum_{i=1}^{N} \Bigg[ & 2x_i^T(t)\mathscr{P}_i^{\alpha}\dot{x}_i(t) + x_i^T(t)\sum_{\beta=1}^{s}\pi_{\alpha\beta}\mathscr{P}_i^{\beta}x_i(t) \\ & - (1-\dot{\tau}_{ij}(t))x_i^T(t-\tau_{ij}(t))\mathscr{Q}_i x_i(t-\tau_{ij}(t)) \\ & + x_i^T(t)\mathscr{Q}_i x_i(t) + \frac{\bar{q}^2}{4}\Gamma_i^T(t)\Sigma_i\mathbb{X}_i^{-1}\Sigma_i^T\Gamma_i(t) \\ & + \frac{\bar{q}^2}{4}\dot{x}_i^T(t)\mathbb{X}_i\dot{x}_i(t) + \Phi_2(t)\zeta_i^T(t)\mathscr{R}_i\zeta_i(t) \\ & - \Phi_1(t)\zeta_i^T(t)\mathscr{R}_i\zeta_i(t) - x_i^T(t)\mathscr{T}_i x_i(t) \\ & + 2x_i^T(t)\mathscr{T}_i x_i(t_k) - x_i^T(t_k)\mathscr{T}_i x_i(t_k) \\ & + \Phi_2(t)\left(2x_i^T(t)\mathscr{T}_i\dot{x}_i(t) - 2\dot{x}_i^T(t)\mathscr{T}_i x_i(t_k)\right) \\ & + \sum_{\substack{j=1\\ j\neq i}}^{N}\Bigg[x_i^T(t)\mathscr{M}_{ji}x_i(t) - (1-\dot{\tau}_{ij}(t)) \\ & (x_j^T(t-\tau_{ij}(t))\mathscr{M}_{ij}x_j(t-\tau_{ij}(t)))\Bigg] \\ & + 2\Big[x_i^T(t)\mathbb{L}_{1i}^T + \dot{x}_i^T(t)\mathbb{L}_{2i}^T\Big]\Big[-\dot{x}_i(t) + A_i^{\alpha}x_i(t) \\ & + \sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij}^{\alpha}x_j(t-\tau_{ij}(t)) + B_i^{\alpha}K_i^{\alpha}x_i(t_k)\Big]\Bigg].\end{aligned} \tag{23}$$

Moreover, the inequality above can be expressed as follows:

$$\mathcal{V}_i(t) \leq \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\left(\Gamma_i^T(t)\bar{\gamma}^{ij}(t)\Gamma_i(t)\right) \tag{24}$$

where $\bar{\gamma}^{ij}(t) = \widehat{\Xi}_{ij} + \bar{\Delta}_{ij}^1 + \bar{\Delta}_{ij}^2$,
$\widehat{\Xi}_{11} = \sum_{\beta=1}^{s}\pi_{\alpha\beta}\mathscr{P}_i^{\beta} + \mathscr{Q}_i - \mathscr{T}_i + \mathscr{M}_{ji} + He\{\mathbb{L}_{1i}^T A_i^{\alpha}\}$,
$\widehat{\Xi}_{12} = \mathscr{T}_i + \mathbb{L}_{1i}^T B_i^{\alpha}K_i^{\alpha}$, $\widehat{\Xi}_{13} = \mathscr{P}_i^{\alpha} + kA_i^{\alpha T}\mathbb{L}_{1i} - \mathbb{L}_{1i}^T$,
$\widehat{\Xi}_{15} = \mathbb{L}_{1i}^T A_{ij}^{\alpha}$, $\widehat{\Xi}_{23} = kK_i^{\alpha T}B_i^{\alpha T}\mathbb{L}_{1i}$,
$\widehat{\Xi}_{33} = He\{-k\mathbb{L}_{1i}^T\} + \frac{\bar{q}^2}{8}He\{\mathbb{X}_i\}$, $\widehat{\Xi}_{35} = k\mathbb{L}_{1i}^T A_{ij}^{\alpha}$,
$\widehat{\Xi}_{44} = -(1-\dot{\tau}_{ij}(t))\mathscr{Q}_i$, $\widehat{\Xi}_{55} = -(1-\dot{\tau}_{ij}(t))\mathscr{M}_{ij}$,
$\bar{\Delta}_{ij}^1 = \Phi_1(t)\Delta_{ij}^1 + \frac{\bar{q}^2}{4}\Sigma_i\mathbb{X}_i^{-1}\Sigma_i^T$, $\bar{\Delta}_{ij}^2 = \Phi_2(t)\Delta_{ij}^2 + \frac{\bar{q}^2}{4}\Sigma_i\mathbb{X}_i^{-1}\Sigma_i^T$ and $\Delta_{ij}^1$, $\Delta_{ij}^2$ and $\Sigma_i$ are mentioned in the theorem statement.

Using schur complenet lemma, convex combination technique and $\mathbb{L}_{2i} = k\mathbb{L}_{1i}$ we obtain $\mathcal{V}_i(t) < 0$ only if $\bar{\gamma}^{ij}(t) < 0$ for all $t \in \left[t_k, t_{k+1}\right)$. Moreover, $w_i(t) \equiv 0$ and integrating $\mathcal{V}_i(t)$ from $t = 0$ to $t = \infty$, we get inequality (6).

Hence by Definition 2 the LIMJS (1) with $w_i(t) \equiv 0$ is SS if $\bar{\gamma}^{ij}(t) < 0$ for all $t \in \left[t_k, t_{k+1}\right)$. Now, we move forward to show that the LIMJS (1) is SS with $\mathscr{H}_\infty$ performance level $\gamma$. For this, $w_i(t) \neq 0$ is assumed and from (1) and $\mathcal{V}_i(t)$, we get

$$
\begin{aligned}
&\mathcal{V}_i(t) + y_i^T(t)y_i(t) - \gamma^2 w_i^T(t)w_i(t) \\
&\leq \sum_{i=1}^{N} \Bigg[ 2x_i^T(t)\mathscr{P}_i^\alpha \dot{x}_i(t) + x_i^T(t) \sum_{\beta=1}^{s} \pi_{\alpha\beta}\mathscr{P}_i^\beta x_i(t) \\
&- (1 - \dot{\tau}_{ij}(t))x_i^T(t - \tau_{ij}(t))\mathscr{Q}_i x_i(t - \tau_{ij}(t)) \\
&+ x_i^T(t)\mathscr{Q}_i x_i(t) + \frac{\bar{q}^2}{4}\Gamma_i^T(t)\Sigma_i\mathbb{X}_i^{-1}\Sigma_i^T\Gamma_i(t) \\
&+ \frac{\bar{q}^2}{4}\dot{x}_i^T(t)\mathbb{X}_i\dot{x}_i(t) + \Phi_2(t)\zeta_i^T(t)\mathscr{R}_i\zeta_i(t) \\
&- \Phi_1(t)\zeta_i^T(t)\mathscr{R}_i\zeta_i(t) - x_i^T(t)\mathscr{T}_i x_i(t) \\
&+ 2x_i^T(t)\mathscr{T}_i x_i(t_k) - x_i^T(t_k)\mathscr{T}_i x_i(t_k) \\
&+ \Phi_2(t)\left(2x_i^T(t)\mathscr{T}_i\dot{x}_i(t) - 2\dot{x}_i^T(t)\mathscr{T}_i x_i(t_k)\right) \\
&+ \sum_{\substack{j=1 \\ j\neq i}}^{N} \Bigg[ x_i^T(t)\mathscr{M}_{ji}x_i(t) - (1 - \dot{\tau}_{ij}(t))(x_j^T(t - \tau_{ij}(t)) \\
&\mathscr{M}_{ij}x_j(t - \tau_{ij}(t)))\Bigg] + 2\Bigg[x_i^T(t)\mathbb{L}_{1i}^T + \dot{x}_i^T(t)\mathbb{L}_{2i}^T\Bigg] \\
&\Bigg[ - \dot{x}_i(t) + A_i^\alpha x_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}^\alpha x_j(t - \tau_{ij}(t)) \\
&+ B_i(r(t))K_i^\alpha x_i(t_k) + D_i^\alpha w_i(t)\Bigg]\Bigg] \\
&+ x_i^T(t)C_i^{\alpha T}C_i^\alpha x_i(t) - \gamma^2 w_i^T(t)w_i(t)
\end{aligned}
$$

Therefore, the above inequality can finally be expressed as

$$
\leq \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} \left(\Gamma_i^T(t)\gamma_{ij}(t)\Gamma_i(t)\right) \tag{25}
$$

where $\gamma_{ij}(t) = \bar{\Xi}_{ij} + \bar{\Delta}_{ij}^1 + \bar{\Delta}_{ij}^2$ and $\bar{\Xi}_{ij}$,$\bar{\Delta}_{ij}^1$ and $\bar{\Delta}_{ij}^2$ are defined earlier.

Utilizing the convex combination technique, if $\gamma_{ij}(t) < 0$ holds for all $t \in [t_k, t_{k+1})$ iff

$$
\bar{\Xi}_{ij} + \bar{\Delta}_{ij}^1 < 0 \tag{26}
$$

$$
\bar{\Xi}_{ij} + \bar{\Delta}_{ij}^2 < 0 \tag{27}
$$

By schur complement (26) and (27) are equivalent to (8) and (9). Hence

$$
\mathcal{V}_i(t) + y_i^T(t)y_i(t) - \gamma^2 w_i^T(t)w_i(t) < 0 \tag{28}
$$

if $\gamma_{ij}(t) < 0$ for all $t \in [t_k, t_{k+1})$.

Integrating (28) from $t = 0$ to $t = \infty$, we obtain the inequality (7), which guarantees the LIMJS (1) is SS with $\mathscr{H}_\infty$ performance level $\gamma$. This concludes the proof. ■

*Theorem 2:* For given scalars $k, 0 < \gamma, 0 < \underline{q} \leq \bar{q}$ and if there exist real matrices $\widehat{\mathscr{P}_i}(r(t)) > 0, \widehat{\mathscr{Q}_i} > 0, \widehat{\mathscr{R}_i} > 0, \widehat{\mathscr{T}_i} > 0, \widehat{\mathbb{X}}_i > 0, \widehat{\mathscr{M}_{ij}} > 0$ and $\mathscr{L}_{1i}, \mathscr{L}_{2i}$, $\mathbb{U}_i^\alpha$ are any matrices with appropriate dimensions, $i, j \in \mathcal{N}$ such that for all $h_k \in \left[\underline{q}, \bar{q}\right]$

$$
\begin{bmatrix} \phi_{ij} + h_k\hat{\Delta}_{ij}^1 & \frac{\bar{q}}{\sqrt{8}}\widehat{\Sigma}_i \\ * & -He\{\widehat{\mathbb{X}}_i\} \end{bmatrix} < 0 \tag{29}
$$

$$
\begin{bmatrix} \phi_{ij} + h_k\hat{\Delta}_{ij}^2 & \frac{\bar{q}}{\sqrt{8}}\widehat{\Sigma}_i \\ * & -He\{\widehat{\mathbb{X}}_i\} \end{bmatrix} < 0 \tag{30}
$$

where
$\phi_{11} = \sum_{\beta=1}^{s}\pi_{\alpha\beta}\widehat{\mathscr{P}_i^\beta} + \widehat{\mathscr{Q}_i} - \widehat{\mathscr{T}_i} + \widehat{\mathscr{M}_{ji}} + He\{A_i^\alpha\mathscr{L}_{1i}\}$,
$\phi_{12} = \widehat{\mathscr{T}_i} + B_i^\alpha\widehat{K}_i^\alpha$, $\phi_{13} = \widehat{\mathscr{P}_i^\alpha} + k\mathscr{L}_{1i}^T A_i^{\alpha T} - \mathscr{L}_{1i}$,
$\phi_{15} = A_{ij}^\alpha\mathscr{L}_{1i}$, $\phi_{17} = D_i^\alpha$, $\phi_{18} = \mathscr{L}_{1i}^T C_i^{\alpha T}$,
$\phi_{23} = k\widehat{K}_i^{\alpha T}B_i^{\alpha T}$, $\phi_{33} = He\{-k\mathscr{L}_{1i}\} + \frac{\bar{q}^2}{8}He\{\widehat{\mathbb{X}}_i\}$, $\phi_{35} = kA_{ij}^\alpha\mathscr{L}_{1i}$, $\phi_{37} = kD_i^\alpha$, $\phi_{44} = -(1 - \tau_{ij})\widehat{\mathscr{Q}_i}$, $\phi_{55} = -(1 - \tau_{ij})\widehat{\mathscr{M}_{ij}}$,$\hat{\Delta}_{11}^1 = -\hat{r}_{11i}$, $\hat{\Delta}_{12}^1 = -\hat{r}_{12i}$, $\hat{\Delta}_{22}^1 = -\hat{r}_{22i}$, $\hat{\Delta}_{11}^2 = \hat{r}_{11i}$, $\hat{\Delta}_{12}^2 = \hat{r}_{12i}$, $\hat{\Delta}_{13}^2 = \widehat{\mathscr{T}_i}$, $\hat{\Delta}_{22}^2 = \hat{r}_{22i}$, $\hat{\Delta}_{23}^2 = -\widehat{\mathscr{T}_i}^T$,
$\widehat{\mathscr{R}_i} = \begin{bmatrix} \hat{r}_{11i} & \hat{r}_{12i} \\ * & \hat{r}_{22i} \end{bmatrix}$, $\widehat{\Sigma}_i = \begin{bmatrix} \hat{r}_{11i} & \hat{r}_{12i} & 0 & 0 & 0 \end{bmatrix}^T$
then, the LIMJS (1) is SS with $\mathscr{H}_\infty$ performance level $\gamma$. Moreover, the MD control gain matrices are obtained from $K_i^\alpha = \mathbb{U}_i^\alpha\left(\mathscr{L}_{1i}\right)^{-1}$

*Proof:*
Define $\mathbb{L}_{1i}^{-1} = \mathscr{L}_{1i}$, $\mathbb{U}_i^\alpha = K_i^\alpha\mathscr{L}_{1i}$, $\widehat{\mathscr{P}_i^\alpha} = \mathscr{L}_{1i}^T\mathscr{P}_i^\alpha\mathscr{L}_{1i}$, $\widehat{\mathscr{Q}_i} = \mathscr{L}_{1i}^T\mathscr{Q}\mathscr{L}_{1i}$, $\widehat{\mathscr{R}_i} = \mathscr{L}_{1i}^T\mathscr{R}\mathscr{L}_{1i}$, $\widehat{\mathscr{T}_i} = \mathscr{L}_{1i}^T\mathscr{T}\mathscr{L}_{1i}$, $\widehat{\mathscr{M}_{ij}} = \mathscr{L}_{1i}^T\mathscr{M}_{ij}\mathscr{L}_{1i}$, and $\widehat{\mathbb{X}}_i = \mathscr{L}_{1i}^T\mathbb{X}_i\mathscr{L}_{1i}$. Now, pre and post multiplying (8) with $diag\{\mathscr{L}_{1i}^T, \mathscr{L}_{1i}^T, \mathscr{L}_{1i}^T, \mathscr{L}_{1i}^T, \mathscr{L}_{1i}^T, \mathscr{L}_{1i}^T, I\}$ and its transpose respectively, which yields (29) and (30). This concludes the proof. ■

## IV. Numerical Example

Consider a LIMJS having three subsystems of the form (5) ($i = 1, 2, 3.$) with two modes $s = 2$, which follows by a Markov process with a TR matrix

$$
\pi = \begin{bmatrix} -1.3 & 1.3 \\ 0.8 & -0.8 \end{bmatrix} \tag{31}
$$

as well as the constant matrices of each subsystems as follows:

**Subsystem 1:**

$$
\begin{aligned}
A_1^1 &= \begin{bmatrix} 0 & 1 \\ -7.91 & -3.94 \end{bmatrix}, A_1^2 = \begin{bmatrix} 0 & 1 \\ -5.51 & -2.69 \end{bmatrix}, \\
A_{12}^1 &= \begin{bmatrix} 0.8 & 0.6 \\ -0.1 & 0.5 \end{bmatrix}, A_{12}^2 = \begin{bmatrix} 0.1 & 0.58 \\ -0.1 & 1 \end{bmatrix}, \\
A_{13}^1 &= \begin{bmatrix} 0.2 & 0 \\ -1 & 0.9 \end{bmatrix}, A_{13}^2 = \begin{bmatrix} 0.2 & 0 \\ -1.4 & 0.3 \end{bmatrix}, \\
B_1^1 &= \begin{bmatrix} 0.2 \\ 0.16 \end{bmatrix}, B_1^2 = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}, \\
C_1^1 &= C_1^2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, D_1^1 = D_1^2 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.
\end{aligned}
$$

**Subsystem 2:**

$$A_2^1 = \begin{bmatrix} 0 & 1 \\ -5.37 & -5.46 \end{bmatrix}, A_2^2 = \begin{bmatrix} 0 & 1 \\ -1.1 & -0.29 \end{bmatrix},$$
$$A_{21}^1 = \begin{bmatrix} 1 & 0.2 \\ 0 & -0.8 \end{bmatrix}, A_{21}^2 = \begin{bmatrix} 1.1 & 0.2 \\ 0.1 & -1.3 \end{bmatrix},$$
$$A_{23}^1 = \begin{bmatrix} 0.8 & 0.4 \\ 1.1 & 1 \end{bmatrix}, A_{23}^2 = \begin{bmatrix} 0.7 & 0.6 \\ 1.1 & -1.1 \end{bmatrix},$$
$$B_2^1 = \begin{bmatrix} 0.5 \\ 2 \end{bmatrix}, B_2^2 = \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix},$$
$$C_2^1 = C_2^2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, D_2^1 = D_2^2 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.$$

**Subsystem 3:**

$$A_3^1 = \begin{bmatrix} 0 & 1 \\ -8.42 & -10.84 \end{bmatrix}, A_3^2 = \begin{bmatrix} 0 & 1 \\ -7.45 & -8.7 \end{bmatrix},$$
$$A_{31}^1 = \begin{bmatrix} 0.5 & 0.1 \\ 0 & -0.3 \end{bmatrix}, A_{31}^2 = \begin{bmatrix} 0.3 & 0.1 \\ 0 & -0.4 \end{bmatrix},$$
$$A_{32}^1 = \begin{bmatrix} -1 & 0 \\ 1.1 & 1.2 \end{bmatrix}, A_{32}^2 = \begin{bmatrix} -1 & 0 \\ 1.1 & 1.3 \end{bmatrix},$$
$$B_3^1 = \begin{bmatrix} 0.5 \\ 2 \end{bmatrix}, B_3^2 = \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix},$$
$$C_3^1 = C_3^2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, D_3^1 = D_3^2 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.$$

By setting, the bounds for interconnection delays are $\tau_{12} = \tau_{21} = 0.52$, $\tau_{13} = \tau_{31} = 0.43$, $\tau_{23} = \tau_{32} = 0.43$ and $k = 0.08$, $\underline{q} = 0.1$ and solving LMIs (29) using Matlab LMI toolbox, it can be obtain that the maximum sampling period $\bar{q} = 0.0979$ and the minimum attenuation performance $\gamma_{min} = 0.0262$, which is listed in Table I and the MD control gain matrices are

$$K_1^1 = \begin{bmatrix} -16.372 & -1.0618 \end{bmatrix}, K_1^2 = \begin{bmatrix} -9.3703 & -2.7314 \end{bmatrix},$$
$$K_2^1 = \begin{bmatrix} -3.7300 & -0.6500 \end{bmatrix}, K_2^2 = \begin{bmatrix} -2.4695 & -0.4272 \end{bmatrix},$$
$$K_3^1 = \begin{bmatrix} -3.7947 & -0.3920 \end{bmatrix}, K_3^2 = \begin{bmatrix} -2.5525 & -0.3843 \end{bmatrix}.$$

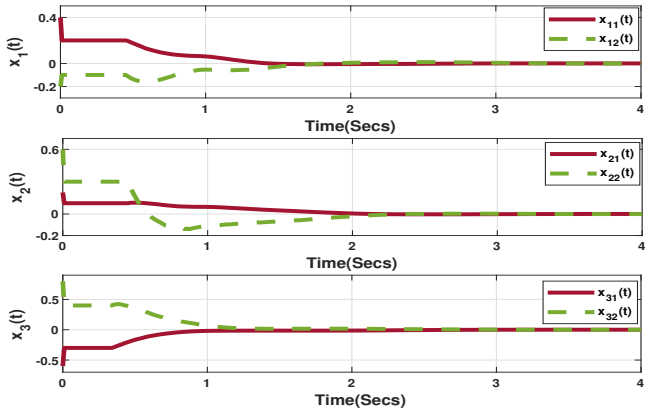



Fig. 1: State response of a subsystems.

For the simulation purpose, let us consider the initial conditions of subsystems $(i = 1, 2, 3)$ are $x_1(t) = \begin{bmatrix} 0.2 & -0.1 \end{bmatrix}^T$, $x_2(t) = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}^T$ and $x_3(t) = \begin{bmatrix} -0.3 & 0.4 \end{bmatrix}^T$, for all $t \in [-h_{ij}, 0]$ with the disturbance input $w_i(t) = e^{-0.05t}(0.005)\sin(2t), (i = 1, 2, 3.), t \geq 0$ and above the control gain matrices. The state responses and control responses of the subsystems $(i = 1, 2, 3.)$ are shown in Fig. 1 and Fig. 2 respectively. Fig. 3 represents the mode evolution of the subsystems $(i = 1, 2, 3)$.

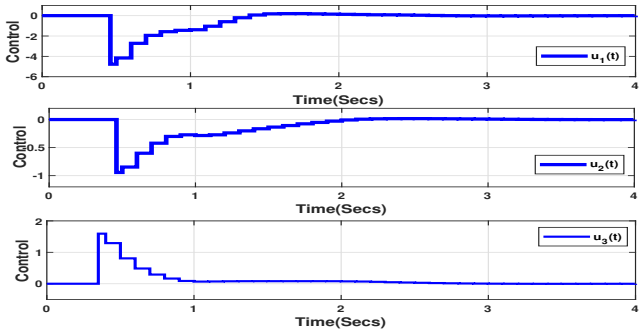



Fig. 2: Control response of a subsystems.

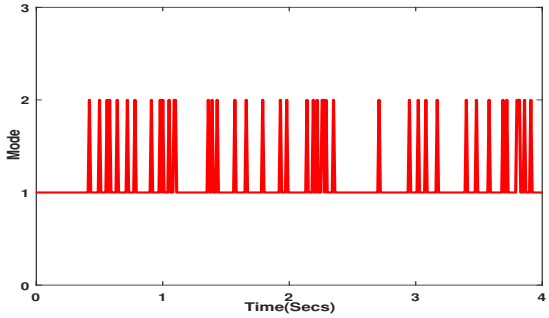



Fig. 3: Response of Markovian jump mode.

TABLE I: Comparison for optimal $\mathscr{H}_\infty$ performance index $\gamma$

| Methods | $\gamma_{min}$ |
|---|---|
| [17] | 1.8177 |
| Theorem 2 | 0.0342 |

Lastly, from comparison Table I, it can be concluded that the proposed Theorem 2 provides minimal $\mathscr{H}_\infty$ performance index when compared to [17]. As a result, this example illustrates the effectiveness of proposed Theorem 2 by comparing the minimum $\mathscr{H}_\infty$ performance index.

*Remark 1:* The authors in [17] discussed the LIMJS with interval MD delays, analyzed the finite-time $\mathscr{H}_\infty$ performance of the interconnected filtering error system without applying any control input. In contrast, this article applies the proposed SDC to the system to achieve the minimized $\gamma_{min}$, as presented in Table I.

## V. CONCLUSION

This paper investigated the SS of LIMJS with time-varying delay in the interconnected term. To evaluate the $\mathscr{H}_\infty$ performance of the LIMJS, a MDSDC was designed. With

the help of the novel LLF, the DDSCs were derived as LMIs to ensure the SS of LIMJS. Finally, the proposed method was numerically validated and compared with existing method to highlight its effectiveness. In the future, by employing novel LLFs, this work can be extended to nonlinear IMJS with mode-dependent time-varying delay, which are unavoidable in real-world situations.

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