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# Adaptive Continuous Terminal Sliding Mode Control for Unmanned Aerial Vehicle

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***Abstract*— This paper introduces an improved control strategy for tracking the altitude and attitude of the quadcopter unmanned aerial vehicle (UAV) in the presence of external disturbance, known as adaptive super-twisting terminal sliding mode control (AST-TSMC). This approach ensures finite-time convergence of the sliding variable and error trajectories using a real second-order sliding mode. An adaptive super-twisting approach is incorporated to reduce the reliance on the upper bound of the unknown disturbance and achieve continuous control. A nonlinear sliding surface is designed to ensure the finite time convergence of the sliding variable and the tracking error. The Lyapunov-based stability analysis is presented to confirm the finite-time convergence. Comparative simulation results of AST-TSMC, ST-TSMC, and TSMC are presented to demonstrate the strategy's performance. The effectiveness of the proposed approach is validated through MATLAB simulation results.**

***Index Terms*— Quadcopter, Super-Twisting, Sliding Mode Control; Adaptive Laws; Lyapunov Stability.**

## I. Introduction

In recent years, the field of UAVs has seen significant growth, with numerous civil and military applications. The inherent characteristics of quadcopters, such as their agility and maneuverability, make them ideal for navigating complex environments where traditional fixed-wing aircraft or helicopters may struggle [1]. However, precise control of quadcopters remains challenging due to their nonlinear dynamics, coupled variables, and sensitivity to external disturbances [2]. Addressing these challenges requires advanced control strategies that can ensure robust and stable performance across a wide range of operating conditions.

Traditional control techniques, such as proportional-integral-derivative (PID) controllers have limitations in handling the nonlinearities and disturbances that affect quadcopters during flight [3]. Researchers have proposed various strategies for quadcopter motion control in the literature, including the Linear Quadratic Regulator (LQR) approach [4], the Model Predictive Control (MPC) method [5]. Another approach to handling the uncertainty/disturbance has been discussed in [6], [7]. Among the various control strategies proposed for quadcopters, SMC has emerged as a promising control design technique due to its inherent robustness against parameter uncertainties, model uncertainties and external disturbances [8], [9]. Also, SMC reduces the system's complexity by decoupling the system dynamics into reduced-order dynamics. However, traditional sliding mode controllers are plagued by the chattering phenomenon, which can degrade system performance and lead to mechanical component wear and tear [10].

To address the limitations of traditional SMC, a Super-Twisting SMC (ST-SMC) [11] algorithm is developed, which incorporates a nonlinear switching term that minimizes chattering effects, thereby enhancing control accuracy and reducing energy consumption. Many literature based on ST-SMC for tracking the quadcopter system can be found in [12]–[14]. However, this approach does not facilitate the choice of flexible gain, which can lead to overestimation and chattering against external disturbance and uncertainty. To address these challenges, an adaptive law-based ST-SMC controller has been devised in [15]–[19]. Another development in the design of SMC can be found in literature as TSMC [20] to ensure the finite time convergence of the sliding variable as well as system trajectories. TSMC has been found application in the design of control for quadcopter UAVs [21], offering superior properties such as fast, finite-time convergence. Furthermore, TSMC has been proposed for tracking the altitude and attitude of a quadcopter [22]. An adaptive TSMC approach was developed for controlling the attitude of a reentry vehicle [23]. Other important literature on the design of AST-TSMC can be found in [24], [25]. In [24], authors proposed the adaptive non singular terminal SMC strategy for the attitude and altitude tracking of quadcopter. A backstepping non-singular terminal sliding mode control with an adaptive algorithm is proposed in [25] for the tracking control of the quadcopter system. Motivated by the existing literature [12], [21], [24], our work proposed the AST-TSMC with terminal sliding variable and an adaptive law to achieve finite time convergence in the presence of external disturbances. The main contributions of this work can be listed as

- The design of AST-TSMC tracking control for quadcopters is proposed in the presence of external disturbance. A terminal sliding surface is designed to achieve finite time convergence of tracking error and sliding variable.
- An adaptive law is chosen to relax the dependency of switching gain on the upper bound of external disturbance and mitigate gain overestimation in the presence of external disturbances.
- To validate the proposed tracking control protocols,

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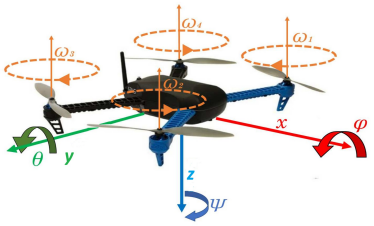



Fig. 1. Schematic of Quadcopter configuration.

MATLAB simulation is presented. Also, a table considering AST-TSMC, ST-TSMC and TSMC is presented to analyze the system performance based on the tracking error and control effort.

## II. QUADCOPTER SYSTEM MODEL

This section presents the nonlinear dynamical model of the quadcopter that is simplified form of quadcopter dynamics using the Newton-Euler approach [21] and is given as:

$$\begin{aligned}
\ddot{x} =& \frac{u_1}{m}(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\\
\ddot{y} =& \frac{u_1}{m}(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)\\
\ddot{z} =& \frac{u_1}{m}(\cos\phi\cos\theta) - g\\
\ddot{\phi} =& a_1\dot{\theta}\dot{\psi} + b_1u_2\\
\ddot{\theta} =& a_2\dot{\phi}\dot{\psi} + b_2u_3\\
\ddot{\psi} =& a_3\dot{\phi}\dot{\theta} + b_3u_4,
\end{aligned} \tag{1}$$

where $[x\ y\ z]^\top$ are the position vectors and $[\phi\ \theta\ \psi]^\top$ are the orientation vectors and are bounded as $-\pi/2 < \phi < \pi/2, -\pi/2 < \theta < \pi/2$ and $-\pi < \psi < \pi$. The $u_1$ is the control input and it is used to control the altitude subsystem. $m$ is the mass of quadcopter and $g$ is the acceleration due to gravity. The $a_1 = (j_y - j_z)/j_x$, $a_2 = (j_z - j_x)/j_y$, $a_3 = (j_x - j_y)/j_z$. $b_1 = 1/j_x, b_2 = 1/j_y, b_3 = 1/j_z$. The $u_2$, $u_3$ and $u_4$ are the control inputs for the roll, pitch and yaw subsystems, respectively. $j_x$, $j_y$, and $j_z$ are the moment of inertia about the $x$, $y$ and $z$ axis, respectively. A schematic of the quadcopter configuration [12] is provided in Fig. 1.

## III. PROBLEM FORMULATION

The quadcopter model (1) can be represented as:

$$\dot{\xi} = \mathbf{f}(\xi) + \mathbf{g}(\xi)\mathbf{u} + \mathbf{d}, \tag{2}$$

where $\xi = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, z, \dot{z}, x, \dot{x}, y, \dot{y}]^\top \in \mathbb{R}^{12}$ is the state vector. $\mathbf{f}(\xi) \in \mathbb{R}^{12}$, $\mathbf{g}(\xi) \in \mathbb{R}^{12\times4}$ and $\mathbf{d} \in \mathbb{R}^{\mathbf{12}}$ are the system matrix, input matrix and external disturbance respectively and the control input is defined as $\mathbf{u} = [u_1, u_2, u_3, u_4]^\top \in \mathbb{R}^4$.

Our main objective is to design a robust controller based on AST-TSMC such that the actual trajectories of altitude and attitude of the quadcopter tracks the desired trajectories within a finite time.

## IV. CONTROLLER DESIGN

### A. Adaptive Super-Twisting Terminal SMC:

In this section, we design the control inputs $u_1$, $u_2$, $u_3$, and $u_4$ for the altitude and attitude tracking of the quadcopter system using an AST-TSMC. For a concise representation of this work, the design steps of $u_2$ for roll tracking system are presented. The other control inputs can be designed similarly. Additionally, for convenience, let the system state variables can be defined as $\boldsymbol{\xi} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6, \xi_7, \xi_8, \xi_9, \xi_{10}, \xi_{11}, \xi_{12}]^\top$.

*1) Attitude Control:*

Now, we design the control input $u_2$ for the roll subsystem from (2) which is given as:

$$\dot{\xi}_1 = \xi_2,\ \ \dot{\xi}_2 = a_1\xi_4\xi_6 + b_1u_2 + d_\phi, \tag{3}$$

where $d_\phi$ represents the bounded disturbance which is differentiable and it is assumed as $d_\phi \le d_{\phi0}$ with $d_{\phi0}$ as its upper bound which is unknown.

Let's define the errors for the roll angle and its rate as:

$$e_\phi = e_{1\phi} = \phi - \phi_d,\ \ e_{2\phi} = \dot{e}_{1\phi} = \dot{\phi} - \dot{\phi}_d, \tag{4}$$

where $\phi_d$ is desired roll angle.

Let us consider the nonlinear sliding surface [21]:

$$s_\phi = e_{2\phi} + p_\phi e_{1\phi} + q_\phi|e_{1\phi}|^{\delta_\phi}\text{sign}(e_{1\phi}), \tag{5}$$

where $p_\phi > 0,\ q_\phi > 0$ and $0 < \delta_\phi < 1$.

The time derivative of the sliding surface $s_\phi$ is defined as

$$\dot{s}_\phi = (a_1\xi_4\xi_6 + b_1u_2 + d_\phi - \ddot{\phi}_d) + p_\phi\dot{e}_{1\phi} + q_\phi\delta_\phi|e_{1\phi}|^{\delta_\phi-1}e_{2\phi}. \tag{6}$$

Next to achieve the stability of the error system (4), an AST-TSMC controller is designed for roll dynamics as:

$$\begin{aligned}
u_2 = -b_1^{-1}\Big(& a_1\xi_4\xi_6 - \ddot{\phi}_d + p_\phi\dot{e}_{1\phi} + q_\phi\delta_\phi|e_{1\phi}|^{\delta_\phi-1}e_{2\phi}\\
&+ \alpha_\phi(t)|s_\phi|^{1/2}\text{sign}(s_\phi) + \frac{1}{2}\int_0^t \beta_\phi(\tau)\text{sign}(s_\phi)d\tau\Big),
\end{aligned} \tag{7}$$

where $\alpha_\phi(t)$ and $\beta_\phi(t)$ are the adaptive gains.

The adaptive gains are updated as [15]:

$$\begin{aligned}
\dot{\alpha}_\phi(t) =& \begin{cases} c_{\phi1}\sqrt{\frac{\gamma_{\phi1}}{2}}\text{sign}(|s_\phi| - \mu_\phi), & \text{if } \alpha_\phi(t) > \alpha_{\phi m}\\ \zeta_\phi, & \text{if } \alpha_\phi(t) \le \alpha_{\phi m}\end{cases}\\
\beta_\phi(t) =& 2\epsilon_\phi\alpha_\phi(t),
\end{aligned} \tag{8}$$

where $\gamma_{\phi1}$, $c_{\phi1}$, $\mu_\phi$, $\zeta_\phi$ and $\epsilon_\phi$ are arbitrary positive constants. The quantity $\alpha_{\phi m}$ is an arbitrary small positive constant.

The finite time stability of the sliding variable is shown in the theorem below.

*Theorem 1:* Consider the roll subsystem (3). If the sliding surface (5), control law (7) and adaptive gain

$$\alpha_\phi(t) > \frac{(2\eta_\phi + \lambda_\phi + 4\epsilon_\phi^2)^2}{12\epsilon_\phi\lambda_\phi} - \frac{\epsilon_\phi(4\eta_\phi + 1)}{\lambda_\phi}$$

satisfy, where $\eta_\phi$, $\lambda_\phi$ and $\epsilon_\phi$ are the positive constants, then the sliding surface and error trajectory converge within a finite time.

*Proof:* Simplifying the (6) and (7), one obtains:

$$\begin{aligned}\dot{s}_\phi =& -\alpha_\phi(t)|s_\phi|^{1/2}\text{sign}(s_\phi) + v_\phi + d_\phi \\ \dot{v}_\phi =& -\frac{\beta_\phi(t)}{2}\text{sign}(s_\phi).\end{aligned} \tag{9}$$

Let us assume the derivative of $d_\phi$ remains bounded as:

$$0 < |\dot{d}_\phi| \leq \eta_\phi;\ \eta_\phi > 0. \tag{10}$$

Suppose $v_\phi^\star = v_\phi + d_\phi$. Then, we can write (9) as:

$$\begin{aligned}\dot{s}_\phi =& -\alpha_\phi(t)|s_\phi|^{1/2}\text{sign}(s_\phi) + v_\phi^\star \\ \dot{v}_\phi^\star =& -\frac{\beta_\phi(t)}{2}\text{sign}(s_\phi) + \dot{d}_\phi.\end{aligned} \tag{11}$$

Let us consider new state variables defined as:

$$w = [w_1\ w_2]^T = [|s_\phi|^{1/2}\text{sign}(s_\phi)\ v_\phi^\star]^T, \tag{12}$$

where $w_1 = |s_\phi|^{1/2}\text{sign}(s_\phi) \Rightarrow |w_1| = |s_\phi|^{1/2}$. We assume that the $\dot{d}_\phi$ remains bounded and is given as:

$$\dot{d}_\phi = \frac{\Delta_\phi(w,t)}{2}\text{sign}(s_\phi) = \frac{\Delta_\phi(w,t)}{2}\frac{w_1}{|w_1|}. \tag{13}$$

From the (10), one can write (13) as:

$$0 < \Delta_\phi(w,t) \leq 2\eta_\phi. \tag{14}$$

The equivalent representation of system (11) becomes

$$\begin{bmatrix}\dot{w}_1 \\ \dot{w}_2\end{bmatrix} = A(w_1)\begin{bmatrix}w_1 \\ w_2\end{bmatrix}, \tag{15}$$

$$\text{where } A(w_1) = \frac{1}{2|w_1|}\begin{bmatrix}-\alpha_\phi(t) & 1 \\ -(\beta_\phi(t) - \Delta_\phi) & 0\end{bmatrix}. \tag{16}$$

Let us consider the Lyapunov function

$$V = V_0(w) + \frac{1}{2\gamma_{\phi1}}(\alpha_\phi(t) - \alpha_\phi^\star)^2 + \frac{1}{2\gamma_{\phi2}}(\beta_\phi(t) - \beta_\phi^\star)^2, \tag{17}$$

where $\gamma_{\phi1}$, $\gamma_{\phi2}$, $\alpha_\phi^\star$ and $\beta_\phi^\star$ are positive constants and

$$V_0(w) = w^T P w;\ P = \begin{bmatrix}\lambda_\phi + 4\epsilon_\phi^2 & -2\epsilon_\phi \\ -2\epsilon_\phi & 1\end{bmatrix} \tag{18}$$

with $\lambda_\phi > 0,\ \epsilon_\phi > 0$ a positive-definite matrix $P$ (see [15]). Taking the time derivative of (17), one obtains

$$\dot{V} = \dot{V}_0(w) + \frac{1}{\gamma_{\phi1}}(\alpha_\phi(t) - \alpha_\phi^\star)\dot{\alpha}_\phi(t) + \frac{1}{\gamma_{\phi2}}(\beta_\phi(t) - \beta_\phi^\star)\dot{\beta}_\phi(t). \tag{19}$$

In view of (18), the first term of (19) can be represented as:

$$\dot{V}_0(w) = w^T[A^T P + PA]w \leq -\frac{1}{2|w_1|}w^T Q w. \tag{20}$$

Symmetric matrix $Q$ is computed taking into account (14) is

$$Q = \begin{bmatrix}Q_{11} & Q_{12} \\ Q_{21} & 4\epsilon_\phi,\end{bmatrix} \tag{21}$$

where

$$\begin{aligned}Q_{11} &= 2\lambda_\phi\alpha_\phi(t) + 4\epsilon_\phi(2\epsilon_\phi\alpha_\phi(t) - \beta_\phi(t)) + 4\epsilon_\phi\Delta_\phi \\ Q_{12} = Q_{21} &= (\beta_\phi(t) - 2\epsilon_\phi\alpha_\phi(t)) - (\lambda_\phi + 4\epsilon_\phi^2) - \Delta_\phi.\end{aligned}$$

As $\beta_\phi(t) = 2\epsilon_\phi\alpha_\phi(t)$, $Q$ is positive-definite with a minimal eigenvalue $2\epsilon_\phi \leq \lambda_\phi\text{min}(Q)$ if

$$\alpha_\phi(t) > \frac{(2\eta_\phi + \lambda_\phi + 4\epsilon_\phi^2)^2}{12\epsilon_\phi\lambda_\phi} - \frac{\epsilon_\phi(4\eta_\phi + 1)}{\lambda_\phi}. \tag{22}$$

By recalling Rayleigh inequality

$$\lambda_{\phi\text{min}}(P)||w||^2 \leq w^T P w \leq \lambda_{\phi\text{max}}(P)||w||^2. \tag{23}$$

It can be observed that

$$\begin{aligned}&||w||^2 = w_1^2 + w_2^2 = |s_\phi| + w_2^2 \Rightarrow |s_\phi|^{1/2} \leq ||w|| \\ &\text{and } |w_1| = |s_\phi|^{1/2} \leq ||w|| \leq \frac{V_0^{1/2}(w)}{\lambda_{\phi\text{min}}^{1/2}(P)}.\end{aligned} \tag{24}$$

From (23) and as $2\epsilon_\phi \leq \lambda_{\phi\text{min}}(Q)$, one obtains (20) as:

$$\dot{V}_0(w) \leq -\frac{\epsilon_\phi}{|w_1|}\frac{V_0(w)}{\lambda_{\phi\text{max}}(P)}. \tag{25}$$

Substituting (24) into (25) gives

$$\dot{V}_0(w) \leq -\tau V_0^{1/2},\ \tau = \frac{\epsilon_\phi\lambda_{\phi\text{min}}^{1/2}(P)}{\lambda_{\phi\text{max}}(P)}. \tag{26}$$

In accordance with (25), one can write (19) as

$$\begin{aligned}\dot{V} &\leq -\tau V_0^{1/2} + \frac{1}{\gamma_{\phi1}}(\alpha_\phi(t) - \alpha_\phi^\star)\dot{\alpha}_\phi(t) + \frac{1}{\gamma_{\phi2}}(\beta_\phi(t) - \beta_\phi^\star)\dot{\beta}_\phi(t) \\ &= -\tau V_0^{1/2} - \frac{c_{\phi1}}{\sqrt{2\gamma_{\phi1}}}|\alpha_\phi(t) - \alpha_\phi^\star| - \frac{c_{\phi2}}{\sqrt{2\gamma_{\phi2}}}|\beta_\phi(t) - \beta_\phi^\star| \\ &\quad + \frac{1}{\gamma_{\phi1}}\dot{\alpha}_\phi(t)(\alpha_\phi(t) - \alpha_\phi^\star) + \frac{c_{\phi1}}{\sqrt{2\gamma_{\phi1}}}|\alpha_\phi(t) - \alpha_\phi^\star| \\ &\quad + \frac{1}{\gamma_{\phi2}}\dot{\beta}_\phi(t)(\beta_\phi(t) - \beta_\phi^\star) + \frac{c_{\phi2}}{\sqrt{2\gamma_{\phi2}}}|\beta_\phi(t) - \beta_\phi^\star|.\end{aligned} \tag{27}$$

In view of (17) and using the inequality $(x_1^2 + y_1^2 + z_1^2)^{1/2} \leq |x_1| + |y_1| + |z_1|$, one obtains:

$$-\tau V_0^{1/2} - \frac{c_{\phi1}}{\sqrt{2\gamma_{\phi1}}}|\alpha_\phi(t) - \alpha_\phi^\star| - \frac{c_{\phi2}}{\sqrt{2\gamma_{\phi2}}}|\beta_\phi(t) - \beta_\phi^\star| \leq -\tau_0 V^{1/2} \tag{28}$$

with $\tau_0 = \min(\tau, c_{\phi1}, c_{\phi2})$. From (28), one can write (27) as:

$$\begin{aligned}\dot{V} &\leq -\tau_0 V^{1/2} + \frac{1}{\gamma_{\phi1}}\dot{\alpha}_\phi(t)(\alpha_\phi(t) - \alpha_\phi^\star) + \frac{1}{\gamma_{\phi2}}\dot{\beta}_\phi(t)(\beta_\phi(t) - \beta_\phi^\star) \\ &\quad + \frac{c_{\phi1}}{\sqrt{2\gamma_{\phi1}}}|\alpha_\phi(t) - \alpha_\phi^\star| + \frac{c_{\phi2}}{\sqrt{2\gamma_{\phi2}}}|\beta_\phi(t) - \beta_\phi^\star|.\end{aligned} \tag{29}$$

Suppose the adaptive law (8) makes the gains $\alpha_\phi(t)$ and $\beta_\phi(t)$ bounded. Then, there exist positive constants $\alpha_\phi^\star$ and $\beta_\phi^\star$ such that $\alpha_\phi(t) - \alpha_\phi^\star < 0$ and $\beta_\phi(t) - \beta_\phi^\star < 0$, $\forall t \geq 0$. Now, one obtains (29) as:

$$\begin{aligned}\dot{V} \leq& -\tau_0 V^{1/2} - |\alpha_\phi(t) - \alpha_\phi^\star|\left(\frac{1}{\gamma_{\phi1}}\dot{\alpha}_\phi(t) - \frac{c_{\phi1}}{\sqrt{2\gamma_{\phi1}}}\right) \\ &- |\beta_\phi(t) - \beta_\phi^\star|\left(\frac{1}{\gamma_{\phi2}}\dot{\beta}_\phi(t) - \frac{c_{\phi2}}{\sqrt{2\gamma_{\phi2}}}\right) = -\tau_0 V^{1/2} + \Xi,\end{aligned} \tag{30}$$

where $\Xi = -|\alpha_\phi(t) - \alpha_\phi^\star|\left(\frac{\dot{\alpha}_\phi(t)}{\gamma_{\phi1}} - \frac{c_{\phi1}}{\sqrt{2\gamma_{\phi1}}}\right) - |\beta_\phi(t) - \beta_\phi^\star|\left(\frac{\dot{\beta}_\phi(t)}{\gamma_{\phi2}} - \frac{c_{\phi2}}{\sqrt{2\gamma_{\phi2}}}\right)$.

**Case 1:** Suppose $|s_\phi| > \mu_\phi$ and $\alpha_\phi(t) > \alpha_{\phi m},\ \forall t \geq 0$. After selecting $\epsilon_\phi = \frac{c_{\phi 2}}{2c_{\phi 1}}\sqrt{\gamma_{\phi 2}/\gamma_{\phi 1}}$, in view of (8), $\dot{\alpha}_\phi(t) = c_{\phi 1}\sqrt{\gamma_{\phi 1}/2}$ and $\dot{\beta}_\phi(t) = c_{\phi 2}\sqrt{\gamma_{\phi 2}/2}$. Then, (30) reduces to

$$\dot{V} \leq -\tau_0 V^{1/2}. \tag{31}$$

**Case 2:** $|s_\phi| \leq \mu_\phi$, then, $\alpha_\phi(t)$ converges as:

$$\dot{\alpha}_\phi(t) = \begin{cases} -c_{\phi 1}\sqrt{\frac{\gamma_{\phi 1}}{2}}, & \text{if } \alpha_\phi(t) > \alpha_{\phi m} \\ \zeta_\phi, & \text{if } \alpha_\phi(t) \leq \alpha_{\phi m} \end{cases} \tag{32}$$

$$\Xi = \begin{cases} 2|\alpha_\phi(t) - \alpha_\phi^\star|\frac{w_1}{\sqrt{2\gamma_{\phi 1}}}, & \text{if } \alpha_\phi(t) > \alpha_{\phi m} \\ -|\alpha_{\phi m} - \alpha_\phi^\star + \zeta_\phi \cdot t|r, & \text{if } \alpha_\phi(t) \leq \alpha_{\phi m}, \end{cases} \tag{33}$$

where $r = (\zeta_\phi/\gamma_{\phi 1} - w_1/\sqrt{2\gamma_{\phi 1}})$. □

As discussed above, the proposed AST-TSMC-based control ensures that both the sliding surface and its derivative remain within certain domain in finite time, even under the external disturbances. Consequently, the tracking of the quadcopter's attitude and altitude is achieved in finite time.

*Theorem 2:* After $s_\phi$ reaches the domain $|s_\phi| \leq \mu_\phi$, it may leave it. However, it is always guaranteed that a real sliding mode (SM) domain $S_\phi$ exists as:

$$S_\phi = \{s_\phi,\ \dot{s}_\phi : |s_\phi| \leq \theta_{\phi 1},\ |\dot{s}| \leq \theta_{\phi 2},\ \theta_{\phi 1} > \mu_\phi\},$$

where $\theta_{\phi 1}$ and $\theta_{\phi 2}$ are the boundaries, $\theta_{\phi 1} > 0$, $\theta_{\phi 2} > 0$.

*Proof:* As the sliding surface enters the domain $|s_\phi| \leq \mu_\phi$, $\alpha_\phi(t)$ satisfies (32). If $\alpha_\phi(t) \leq \alpha_{\phi m}$, $\alpha_\phi(t)$ will start increasing such that $\alpha_\phi(t) = \alpha_{\phi m} + \zeta_\phi \cdot t$. Consequently, $|s_\phi|$ may become greater than $\mu_\phi$ as the gains $\alpha_\phi(t)$ and $\beta_\phi(t)$ decrease. Thus, it is assured that $s_\phi$ always remains in larger domain $|s_\phi| \leq \theta_{\phi 1},\ \theta_{\phi 1} > \mu_\phi$ [15].

In view of (8) and (11), if $|s_\phi| \leq \mu_\phi$ and $|\dot{s}_\phi|$ hold, then

$$|\dot{s}_\phi| \leq (\alpha_\phi(T_{\phi 1}))\mu^{1/2} + [\epsilon_\phi \alpha_\phi(T_{\phi 1}) + \eta_\phi](T_{\phi 2} - T_{\phi 1}) = \bar{\theta}_{\phi 2},$$

where $T_{\phi 1}$ and $T_{\phi 2}$ denote the time instants when $s_\phi$ enters the domain $|s_\phi| \leq \mu_\phi$ and leaves this domain afterwards, respectively. When $|s_\phi|$ satisfies $\mu_\phi < |s_\phi| \leq \theta_{\phi 1}$, then

$$\begin{aligned} |\dot{s}_\phi| \leq & (\theta_{\phi 1}^{1/2} + \epsilon_\phi)(\alpha_\phi(T_{\phi 2}) + c_{\phi 1}\sqrt{\theta_{\phi 1}\eta_\phi/2})(T_{\phi 3} - T_{\phi 2}) \\ & + \eta_\phi(T_{\phi 3} - T_{\phi 2}) = \tilde{\theta}_{\phi 2}, \end{aligned}$$

where $T_{\phi 2}$ and $T_{\phi 3}$ ($T_{\phi 3} > T_{\phi 2}$) represent the time instant when $s_\phi$ leaves the domain $|s_\phi| \leq \mu_\phi$ and enters this domain thereafter, respectively. Then, the real SM domain becomes

$$S_\phi = \{s_\phi,\ \dot{s}_\phi : |s_\phi| \leq \theta_{\phi 1},\ |\dot{s}_\phi| \leq \theta_{\phi 2},\ \theta_{\phi 1} > \mu_\phi\},$$

where $\theta_{\phi 2} = \max(\bar{\theta}_{\phi 2},\ \tilde{\theta}_{\phi 2})$. □

Now, the boundedness of the adaptive gains $\alpha_\phi(t)$ and $\beta_\phi(t)$ is given in *Proposition 1*.

*Proposition 1:* The $\alpha_\phi(t)$ and $\beta_\phi(t)$ remain bounded.

*Proof:* For region $\mu_\phi < |s_\phi| \leq \theta_{\phi 1}$, a solution of (8) is

$$\alpha_\phi(t) = \alpha_\phi(0) + c_{\phi 1}\sqrt{\gamma_{\phi 1}/2} \cdot t,\ 0 \leq t \leq t_f, \tag{34}$$

where $t_f$ is the finite reaching time. For $|s_\phi| \leq \mu_\phi$, the gains $\alpha_\phi(t)$ and $\beta_\phi(t)$ are decreasing. Thus, $\alpha_\phi(t)$ is bounded which results in bounded $\beta_\phi(t)$ in the real 2-SM. □

*Proposition 2:* In view of (22), the AST-TSMC controller (7) forces $s_\phi$ and its derivative to a domain $S_\phi$ in finite time which can be expressed as:

$$t_f \leq 2V^{1/2}(0)/\zeta_0,\ \zeta_0 = \min(\tau, c_{\phi 1}, c_{\phi 2}) \tag{35}$$

*Proof:* The inequality (22) is satisfied in finite time provided its right-hand side remains bounded and $\alpha_\phi(t)$ is increasing linearly with time in view of (8). The inequality (35) is obtained by directly integrating inequality (30) taking into consideration that $\Xi = 0$. For $\mu_\phi,\ s_\phi,\ \dot{s}_\phi \to S_\phi(s_\phi, \dot{s}_\phi)$ in finite time. □

*Remark 1:* By using an adaptive law, the proposed controller enhances system performance by providing robustness against disturbances, reducing chattering, and continuously updating control gains to adapt to time-varying parameters, thereby mitigating disturbances and reducing gain overestimation.

By applying the similar approach, we design nonlinear sliding surfaces and adaptive laws for the pitch ($\theta$) and yaw ($\psi$) subsystems. The designed sliding surfaces are as follows:

$$\begin{aligned} s_\theta =& e_{2\theta} + p_\theta e_{1\theta} + q_\theta |e_{1\theta}|^{\delta_\theta} \text{sign}(e_{1\theta});\ e_{1\theta} = e_\theta \\ s_\psi =& e_{2\psi} + p_\psi e_{1\psi} + q_\psi |e_{1\psi}|^{\delta_\psi} \text{sign}(e_{1\psi});\ e_{1\psi} = e_\psi, \end{aligned} \tag{36}$$

where all the constants are appropriately designed as shown for the sliding variable $s_\phi$.

The adaptive gains are updated as:

$$\begin{aligned} \dot{\alpha}_\theta(t) =& \begin{cases} c_{\theta 1}\sqrt{\frac{\gamma_{\theta 1}}{2}}\text{sign}(|s_\theta| - \mu_\theta), & \text{if } \alpha_\theta(t) > \alpha_{\theta m} \\ \zeta_\theta, & \text{if } \alpha_\theta(t) \leq \alpha_{\theta m} \end{cases} \\ \beta_\theta(t) =& 2\epsilon_\theta \alpha_\theta(t) \\ \dot{\alpha}_\psi(t) =& \begin{cases} c_{\psi 1}\sqrt{\frac{\gamma_{\psi 1}}{2}}\text{sign}(|s_\psi| - \mu_\psi), & \text{if } \alpha_\psi(t) > \alpha_{\psi m} \\ \zeta_\psi, & \text{if } \alpha_\psi(t) \leq \alpha_{\psi m} \end{cases} \\ \beta_\psi(t) =& 2\epsilon_\psi \alpha_\psi(t), \end{aligned} \tag{37}$$

where $c_{\theta 1,\psi 1}$, $\gamma_{\theta 1,\psi 1}$, $\mu_{\theta,\psi}$, $\zeta_{\theta,\psi}$ and $\epsilon_{\theta,\psi}$ are arbitrary positive constants. The $\alpha_{\theta m,\psi m}$ is an arbitrary small positive constant.

With these designed sliding surfaces and adaptive laws, the AST-TSMC for $\theta$, and $\psi$ are as follows:

$$\begin{aligned} u_3 =& -b_2^{-1}\Big(a_2\dot{\phi}\dot{\psi} - \ddot{\theta}_d + p_\theta \dot{e}_{1\theta} + q_\theta \delta_\theta |e_{1\theta}|^{\delta_\theta - 1} e_{2\theta} \\ & + \alpha_\theta(t)|s_\theta|^{1/2}\text{sign}(s_\theta) + \frac{1}{2}\int_0^t \beta_\theta(\tau)\text{sign}(s_\theta)d\tau\Big) \\ u_4 =& -b_3^{-1}\Big(a_3\dot{\phi}\dot{\theta} - \ddot{\psi}_d + p_\psi \dot{e}_{1\psi} + q_\psi \delta_\psi |e_{1\psi}|^{\delta_\psi - 1} e_{2\psi} \\ & + \alpha_\psi(t)|s_\psi|^{1/2}\text{sign}(s_\psi) + \frac{1}{2}\int_0^t \beta_\psi(\tau)\text{sign}(s_\psi)d\tau\Big), \end{aligned} \tag{38}$$

where all the constants are appropriately designed as shown for the control $u_2$.

*2) Altitude Control:*

By the similar approach as previous, we design nonlinear sliding surface, adaptive law and control for the $z$ subsystem. The designed sliding surface is given as:

$$s_z = e_{2z} + p_z e_{1z} + q_z |e_{1z}|^{\delta_z} \text{sign}(e_{1z});\ e_{1z} = e_z, \tag{39}$$

where $p_z > 0$, $q_z > 0$ and $0 < \delta_z < 1$.
The adaptive gain is updated as:

$$\begin{aligned}\dot{\alpha}_z(t) =& \begin{cases} c_{z1}\sqrt{\frac{\gamma_{z1}}{2}}\text{sign}(|s_z| - \mu_z), \text{if } \alpha_z(t) > \alpha_{zm} \\ \zeta_z, \qquad\qquad \text{if } \alpha_z(t) \leq \alpha_{zm} \end{cases} \\ \beta_z(t) =& 2\epsilon_z\alpha_z(t),\end{aligned} \tag{40}$$

where $c_{z1}$, $\gamma_{z1}$, $\mu_z$, $\zeta_z$ and $\epsilon_z$ are arbitrary positive constants. The quantity $\alpha_{zm}$ is an arbitrary small positive constant.
With these designed sliding surface and adaptive law, the AST-TSMC for $z$ is as follows:

$$\begin{aligned}u_1 = \frac{m}{(\cos\phi\cos\theta)}\Bigg(& g + \ddot{z}_d - p_z\dot{e}_{1z} - q_z\delta_z|e_{1z}|^{\delta_z - 1}e_{2z} \\ & - \alpha_z(t)|s_z|^{1/2}\text{sign}(s_z) - \frac{1}{2}\int_0^t \beta_z(\tau)\text{sign}(s_z)d\tau\Bigg),\end{aligned} \tag{41}$$

where all the constants are appropriately designed as given for the control $u_2$.

## V. SIMULATION

This section presents the MATLAB simulation results to present the effectiveness of the proposed design approach. The quadcopter parameters values are selected as $j_x = j_y = 2.9125 \times 10^{-2}Nm$, $j_z = 5.5225 \times 10^{-2}Nm$, $m = 1.5kg$ and $g = 9.81m/sec^2$. The design parameters has been selected as $p_{\phi,\theta,\psi,z} = 0.05$, $q_{\phi,\theta,\psi,z} = 0.01$ and $\delta_{\phi,\theta,\psi,z} = 0.5$, $c_{\phi 1,\theta 1,\psi 1,z1} = 20$, $\gamma_{\phi 1,\theta 1,\psi 1,z1} = 2$, $\mu_{\phi,\theta,\psi,z} = 0.01$, $\zeta_{\phi,\theta,\psi,z} = 0.01$ and $\epsilon_{\phi,\theta,\psi,z} = 1$, $\alpha_{\phi m,\theta m,\psi m,zm} = 0.5$. The switching gain for the $ST-TSMC$ and the $TSMC$ has been selected as $k1_{ST-TSMC} = 5$, $k2_{ST-TSMC} = 2$ and $k_{TSMC} = 0.5$. The desired trajectories are given as $z_d(t) = 5 + 0.01\cos(t)$, $\phi_d = 10(\pi/180)\cos(0.1t)$, $\theta_d = 10(\pi/180)\cos(0.5t)$ and $\psi_d = 10(\pi/180)\cos(0.2t)$. The initial conditions are chosen as $\boldsymbol{\xi} = [0,0,0,0,0,0,0,0,0,0,0,0]^\top$. The external disturbances $d(t) = 0.03\sin(0.1t)$ is considered for roll, pitch, yaw and z subsystems. The evolution of the tracking trajectories has been demonstrated in the Fig. 2. The growth of tracking error has been presented in the Fig. 3. The evolution of the sliding variable and the control effort has been presented in the Figs. 4 and 5. To analyse a comparative notion among the AST-TSMC, ST-TSMC, TSMC, we have done the simulation considering all the three control design method. Although, almost exact tracking performance can be achieved with three control strategies but from the Fig. 5, it can be observed that the the reduced chattering, and continuous control can be achieved using AST-TSMC compare to ST-TSMC and TSMC.

To analyze the system performance a few indices have been considered comparing AST-TSMC, ST-TSMC and TSMC in Table I. The indices are chosen in [26] as follows

- Total Tracking error, $e_{\phi,\theta,\psi,z} = \int_0^{t_f} \|e_{\phi,\theta,\psi,z}\| d\tau$,
- Total control effort, $u_{2,3,4,1} = \int_0^{t_f} \|u_{2,3,4,1}\| d\tau$.

From Table I, it can be observed that the norm of average tracking error is almost in the same range, but the control effort required for the AST-TSMC is less compared to the considered ST-TSMC and TSMC. In a way, it can be mentioned that the effort of the controller in adaptive SMC is relatively relaxed, which again depends on the judicious choice of adaptive law design parameters and switching gain design of the ST-TSMC and TSMC . Also, due to the particular structure of adaptive law, there is flexibility in the control gain design parameters.

TABLE I
COMPARING PERFORMANCE OF AST-TSMC, ST-TSMC AND TSMC

| Controller | Total Tracking Error | Total Control Effort |
|---|---|---|
| AST-TSMC | 41.66 | 262.55 |
| ST-TSMC | 40.72 | 264 |
| TSMC | 40.73 | 265.26 |

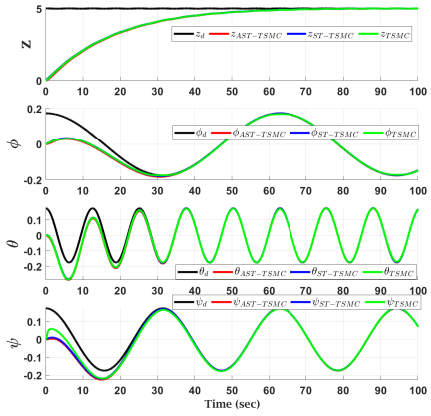



Fig. 2. Tracking trajectories of roll ($\phi$), pitch ($\theta$), yaw ($\psi$) and altitude($z$).

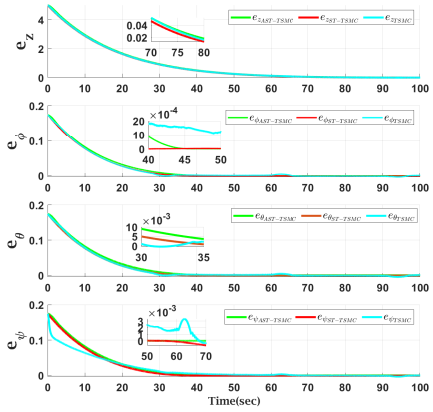



Fig. 3. Errors associated with roll ($\phi$), pitch ($\theta$), yaw ($\psi$) and altitude($z$).

## VI. CONCLUSION

This paper details the development and implementation of AST-TSMC strategy for quadcopter altitude and attitude tracking in the presence of external disturbance. This proposed method can achieved finite time convergence with continuous control and relaxation of dependance of switching gain on external bounded disturbance. The Lyapunov-based

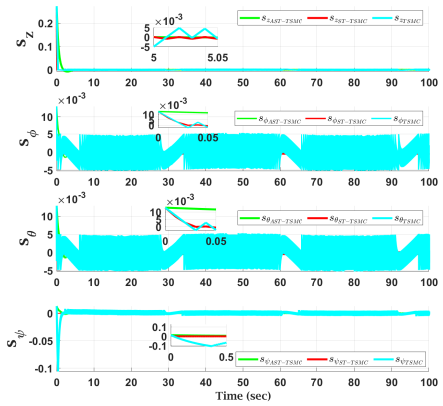



Fig. 4. Sliding variables of roll ($\phi$), pitch ($\theta$), yaw ($\psi$) and altitude($z$).

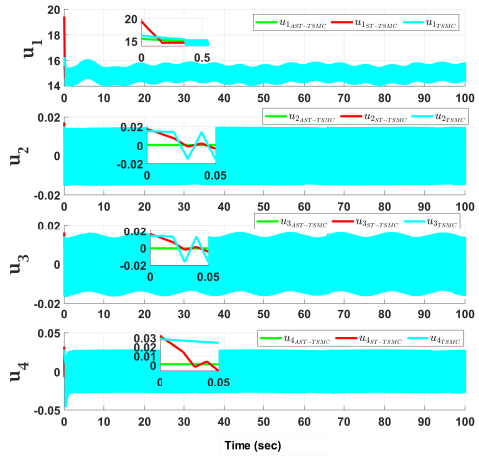



Fig. 5. The control effort for roll ($\phi$), pitch ($\theta$), yaw ($\psi$) and altitude($z$).

stability analysis ensures finite-time convergence. Finally, simulation results validate the effectiveness of the proposed approach. The comparative simulation results have been shown to demonstrate the effectiveness of the AST-TSMC, ST-TSMC, and TSMC. To ensure the fast convergence and the hardware implementation, the AST-TSMC can be considered as the future scope of the work.

## REFERENCES


[1] D. Mellinger, N. Michael, and V. Kumar, "Trajectory generation and control for precise aggressive maneuvers with quadrotors," *The International Journal of Robotics Research*, vol. 31, no. 5, pp. 664–674, 2012.

[2] Y. Zou and Z. Meng, "Immersion and invariance-based adaptive controller for quadrotor systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 11, pp. 2288–2297, 2018.

[3] A. Tayebi and S. McGilvray, "Attitude stabilization of a vtol quadrotor aircraft," *IEEE Transactions on control systems technology*, vol. 14, no. 3, pp. 562–571, 2006.

[4] F. Rinaldi, S. Chiesa, and F. Quagliotti, "Linear quadratic control for quadrotors uavs dynamics and formation flight," *Journal of Intelligent & Robotic Systems*, vol. 70, pp. 203–220, 2013.

[5] A. T. Hafez, A. J. Marasco, S. N. Givigi, M. Iskandarani, S. Yousefi, and C. A. Rabbath, "Solving multi-uav dynamic encirclement via model predictive control," *IEEE Transactions on control systems technology*, vol. 23, no. 6, pp. 2251–2265, 2015.

[6] S. Kumar, S. K. Soni, A. Kumar Pal, S. Kamal, and X. Xiong, "Nonlinear polytopic systems with predefined time convergence," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 71, no. 2, pp. 632–636, 2024.

[7] S. Kumar, S. K. Soni, and S. Kamal, "Predefined-time frequency estimation of biased sinusoidal signals," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 71, no. 4, pp. 2369–2373, 2024.

[8] Y. Shtessel, C. Edwards, L. Fridman, A. Levant *et al.*, *Sliding mode control and observation*. Springer, 2014, vol. 10.

[9] S. K. Soni, S. Kumar, S. Wang, A. Sachan, D. Boutat, and D. Geha, "Sliding mode even-triggered tracking control for robot manipulators with state constrains," *IECON 2022–48th Annual Conference of the IEEE Industrial Electronics Society*, pp. 1–6, 2022.

[10] J.-J. E. Slotine, W. Li *et al.*, *Applied nonlinear control*. Prentice hall Englewood Cliffs, NJ, 1991, vol. 199, no. 1.

[11] A. Levant, "Sliding order and sliding accuracy in sliding mode control," *International journal of control*, vol. 58, no. 6, pp. 1247–1263, 1993.

[12] A. Yesmin, K. Kumari, A. V. Borkar, A. Sinha, and H. Arya, "Software-in-the-loop validation of super twisting based sliding mode control for quadcopters," *IFAC-PapersOnLine*, vol. 55, no. 22, pp. 123–128, 2022.

[13] D. Matouk, F. Abdessemed, O. Gherouat, and Y. Terchi, "Second-order sliding mode for position and attitude tracking control of quadcopter uav: Super-twisting algorithm," *International Journal of Innovative Computing, Information and Control*, vol. 16, no. 1, pp. 29–43, 2020.

[14] S. K. Soni, S. Singh, S. Kumar, H. S. Sahu, and R. Saket, "Solar-to-vehicle charging with maximum power point tracking using super-twisting controller," *2023 IEEE International Transportation Electrification Conference (ITEC-India)*, pp. 1–6, 2023.

[15] Y. Shtessel, M. Taleb, and F. Plestan, "A novel adaptive-gain supertwisting sliding mode controller: Methodology and application," *Automatica*, vol. 48, no. 5, pp. 759–769, 2012.

[16] S. Rajappa, C. Masone, H. H. Bülthoff, and P. Stegagno, "Adaptive super twisting controller for a quadrotor uav," *2016 IEEE International Conference on Robotics and Automation (ICRA)*, pp. 2971–2977, 2016.

[17] S. Kumar, R. K. Sharma, and S. Kamal, "Adaptive super-twisting guidance law with extended state observer," *IECON 2021–47th Annual Conference of the IEEE Industrial Electronics Society*, pp. 1–6, 2021.

[18] S. Kumar, S. K. Soni, A. Sachan, S. Kamal, and B. Bandyopadhyay, "Adaptive super-twisting guidance law: an event-triggered approach," *2022 16th International Workshop on Variable Structure Systems (VSS)*, pp. 190–195, 2022.

[19] S. Kumar, "Rated observation and control for a class of dynamical systems," Ph.D. dissertation, IIT (BHU), 2023.

[20] Y. Wu, X. Yu, and Z. Man, "Terminal sliding mode control design for uncertain dynamic systems," *Systems & Control Letters*, vol. 34, no. 5, pp. 281–287, 1998.

[21] V. K. Tripathi, A. K. Kamath, L. Behera, N. K. Verma, and S. Nahavandi, "An adaptive fast terminal sliding-mode controller with power rate proportional reaching law for quadrotor position and altitude tracking," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 6, pp. 3612–3625, 2021.

[22] R. Fessi, S. Bouallègue, J. Haggège, and S. Vaidyanathan, "Terminal sliding mode controller design for a quadrotor unmanned aerial vehicle," *Applications of sliding mode control in science and engineering*, pp. 81–98, 2017.

[23] B. An, B. Wang, Y. Wang, and L. Liu, "Adaptive terminal sliding mode control for reentry vehicle based on nonlinear disturbance observer," *IEEE Access*, vol. 7, pp. 154 502–154 514, 2019.

[24] H. Ghadiri, M. Emami, and H. Khodadadi, "Adaptive super-twisting non-singular terminal sliding mode control for tracking of quadrotor with bounded disturbances," *Aerospace Science and Technology*, vol. 112, p. 106616, 2021.

[25] A. Modirrousta and M. Khodabandeh, "Adaptive non-singular terminal sliding mode controller: new design for full control of the quadrotor with external disturbances," *Transactions of the Institute of Measurement and Control*, vol. 39, no. 3, pp. 371–383, 2017.

[26] J. Bae and Y. Kim, "Adaptive controller design for spacecraft formation flying using sliding mode controller and neural networks," *Journal of the Franklin Institute*, vol. 349, no. 2, pp. 578–603, 2012.