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Dynamic Data-based Modeling for Froth Flotation Process

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Abstract - **Froth flotation is a widely used technique for mineral separation, but it poses significant challenges for modeling due to the complex phenomena that happen during the process. To account for the implementation of optimization and control-related studies in flotation systems, this work deals with the development of a digital twin or surrogate to the original model for the froth flotation system. Initially, a first-principles, three-phase model having 61 equations (7 Ordinary Differential Equations and 54 nonlinear equations) was considered and implemented. The model consists of four inputs, a set of initial conditions for feed, one primary output, and sixteen intermediate outputs. 15,000 time-series data were sampled from the physics-based model to initiate the data-driven modeling. The well-known recurrent neural network variants, namely RNN, LSTM, and GRU, were developed as potential surrogates for the three-phase model by following a multi-input and single-output approach. On performing a comparison study for all the output predictions on test data, it was found that GRU outperforms LSTM and RNN for a majority of the outputs (9 out of 17), especially for highly dynamic ones. Also, GRU consists of 25% fewer model parameters than LSTM networks, which thereby decreases the time for training and reduces the load on the optimizer. Overall, this study provides a deep understanding of froth flotation dynamics and establishes the efficacy of advanced neural network architectures in enhancing predictive modeling in mineral separation processes.**

Keywords: Froth flotation, three-phase model, time-series data, recurrent neural networks, and model parameters.

1. INTRODUCTION

Froth flotation is the selective separation technique of minerals from their ores based on the density difference and the hydrophobic nature of the minerals. This technique is widely employed in many fields, including recovering valuable minerals such as copper, lead, zinc, nickel, and sulfide ore. For instance, it is used to separate coal from ash-forming minerals, de-ink wastepaper for reusing, and other materials [1]. The flotation is a complex process comprising chemistry, equipment, and operational components [1]. Nonetheless, flotation is a dynamic process, and a physics-based model containing the dynamics at each stage is found to be challenging. Subsequently, there is a need for the usage of reduced-order models, like digital twins, for optimization and control studies to achieve better recovery of minerals with low operation costs. In this regard, significant research was done in building empirical and phenomenological models [2].

Empirical models are based on experimental data, and the model relates the dependent and independent variables using a suitable mathematical equation. However, the parameters obtained by such analysis usually do not have any physical significance [3]. Phenomenological models are derived from the conservation of mass, momentum, and energy equations [3 - 4]. Phenomenological models typically involve complex, nonlinear, and non-homogeneous differential equations [5]. Solving these intricate equations is challenging, necessitating numerous assumptions and precise initial conditions for real-world applicability.

Moreover, it is a computationally expensive study. To the best of our knowledge, there is no suitable model for describing the complete flotation process under a wide range of reagent conditions [5 - 6]. On the other hand, data-driven models such as FNN (Feed Forward Neural Network), RNN (Recurrent Neural network), CNN (Convolution Neural Network), LSTM (Long Short-Term Memory), GRU (Gated Recurrent Unit), etc., could be considered a promising alternative for capturing the dynamics of the complex process of flotation without relying heavily on prior knowledge of physical laws [7 - 9].

While the data-based models share similarities with empirical models in their reliance on data, they go beyond empirical relationships by utilizing sophisticated algorithms and architectures to extract meaningful information from the data. Despite these advantages, data-based models face their own set of challenges. Limited availability of experimental data hampers model training and validation efforts, often resulting in suboptimal performance [7 - 9]. Thus, this study initially aims to utilize the model based on a continuous system to understand the process variables and parameters in depth. Later, it focuses on developing dynamic data-based models using RNN, LSTM, and GRU for the considered input-output data to emphasize the limitations posed by sparse experimental data and computational complexities.

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2. LITERATURE REVIEW

Modeling the froth phase is crucial yet essential for better prediction of recovery and grades as it provides the relationship between the quantity of minerals in the froth and the concentrate. Quintanilla et al. recently developed a new dynamic flotation model of a single flotation cell based on the three phases, pulp phase, froth phase, and pulp-froth interface, for the control purpose [10]. The model assumed the flotation cell as a well-mixed, continuously stirred tank reactor in the pulp phase. Moreover, the feed flow rate, head grade, fraction of solids and bubble size classes in the pulp, particle sizes for the different species, and solids density were assumed to be known [10]. The proposed model was a three-phase dynamic model for a single mineral, which includes the assessments of froth stability via air recovery, bursting rate, and bubble coalescence model equations, in addition to the pulp-froth interface. The dynamic model was a high-index differential and algebraic system consisting of 61 equations with four inputs and, overall, seventeen outputs [10]. The model was simulated to perform the sensitivity analysis of the fitting parameters. The parameters include a, b, and c in the bursting rate (v_b), f in the froth recovery factor (R_f), n and C in the overflow bubble size in froth ($d_{b,\ froth\ out}$), and parameter K that corresponds to number of bubble size class in the pulp. Among all the seven parameters, it was concluded that parameter *n* of the overflow bubble size and parameters a and c of the bursting rate (v_b) were the most sensitive parameters.

Modelling the entire flotation process using the first principles is a challenging task, and a complex nonlinear partial differential equation system is not yet sufficient to represent the behaviour of the process [7]. Thus, data-based models such as LSTM networks, Convolutional Neural Networks (CNNs), Decision Trees, etc., are the need of the hour. Recently, many researchers have shown their attention to applying such models in flotation studies [2, 7 – 9]. One such application is the FlotationNet model developed by Y. Pu et al. [7]. This work focuses on the extraction of minerals from iron ore, as it allows for the separation of unwanted silica particles from the desired minerals. However, data scarcity is a crucial issue restricting FlotationNet's performance, and thus, data augmentation could be a straightforward measure to enhance the model's performance [7]. The tunning of the hyperparameters, such as the time lag for the LSTM input sequence, number of layers, number of units in each and learning rate, could be performed to obtain the optimal hyperparameters.

Zhang et al. used machine vision to extract the essential features of the froth to predict the desired grade [9]. Using those features, the authors developed the encoder-decoder model for predicting the zinc tailing in the lead–zinc flotation plant, which consisted of three-stage roughers, three-stage cleaners, and three-stage scavengers [9]. The proposed model leveraged the feature time series of the first rougher and previously measured tailings grades to predict the current tailings grade. However, the model's accuracy depends on the quality of the previously measured grade information, which could be a potential limitation. Another application of the data-based model in the flotation process is the predictive machine learning model developed by Costa et al., which aimed to eliminate periodic sample material and the delay associated with laboratory analysis of the flotation process [8].

These studies highlight the potential of deep learning models in capturing the complex relationships between process variables and product quality while also emphasizing the need for a large amount of data and the careful feature selection and model configuration to enhance the model's performance. From the study of the above works, it can be found that one of the significant problems in developing a suitable data-based model for a complex flotation process is both the quantity and quality of the data. Moreover, selecting the input features and the appropriate model, such as RNN, LSTM, GRU, CNN, FNN and so on, with optimal hyperparameters, is challenging due to computational constraints. Hence, in this study, the primary focus lies in choosing and solving an appropriate physics-based model from the literature, using which voluminous data could be sampled. Since, in a practical scenario, it is challenging to obtain such large amounts of data at a lab scale, the first-principles-based model, which is computationally expensive, is considered for this study. The sequence of the work performed in this study is given below.

1. First-principles-based Model Selection, Implementation and Execution.
2. Time series Data sampling using the considered model.
3. Development of dynamic data-based models such as RNN, LSTM, and GRU, which are the potential surrogates of the considered model, for usage in optimization and control-related studies.
4. Comparison study of the developed recurrent neural network models.

3. METHODOLOGY

3.1 First-principles-based model

Owing to its applicability, the physics-based model given in the literature by Quintanilla et al. [10], namely the three-phase model, is considered. The three-phase model has advantages over the other kinetic models, such as the introduction of the interface between the pulp and froth phase, which gives the idea of the stability of the froth, which is directly linked with the recovery of air as well as the mineral recovery. The model consists of a system of differential-algebraic equations (DAEs) that need to be solved simultaneously. The system of the DAE is converted into an initial value problem and solved using the explicit 8^{th}-order Runge Kutta method in Python 3.8.17. While solving the system of DAEs, the values of the parameters, operating conditions, and feed properties are taken from the literature [11]. There are 61 equations, including five nonlinear differential equations, two linear differential equations, and

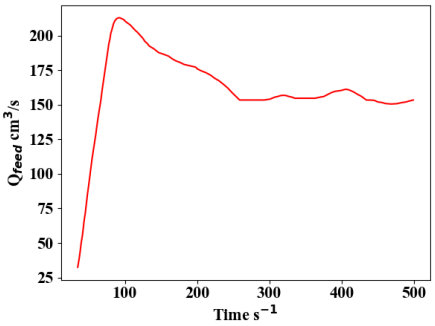


Fig. 1. Variation of the feed rate

54 algebraic systems of equations in the model. Among them, five nonlinear differential equations, two linear differential equations and 49 algebraic systems of equations were used to solve simultaneously for 0 to 500 sec.

The initial values of some of the variables, such as the pulp height, gas holdup, and mineral mass considered, were taken from literature, like $d_{b,\ froth\ out}$, Q_{conc}, the height of the pulp, and froth air recovery. In contrast, those for gas holdup and the mass of minerals in the cell were manually selected. In addition, the feed rate, which varied with time, is considered one of the input variables. The feed rate variation is taken from the literature [11] and is shown in Fig. 1. Also, tailing flowrate ($Q_{tailing}$), inlet air flowrate ($Q_{air,\ in}$) and mineral mass flowrate in the feed (m_{feed}) are considered as additional inputs. The features like actual pulp height (h_p), the height of the froth (h_f), gas velocity out of pulp (V_{gout}), the mineral concentration of pulp ($C_{tailing}$), mineral mass in the pulp (M), overflowing bubble size ($d_{b,\ froth\ out}$), interphase bubble size ($d_{b,\ int}$), interphase velocity (v^*_g), gas hold up (ϵ_0^t), concentrate flowrate (Q_{conc}), rate constant (k), bubble surface area flux(s_b), mass flowrate due to entrainment (m_{ENT}), mass flowrate due to true flotation (m_{TF}), tailing mass flowrate ($m_{tailing}$), froth air recovery (α^*), and mineral recovery (R) (primary output) are considered as the 17 output variables.

3.2 Data Sampling and Pre-processing

A random Gaussian signal is considered to be the input sequential data. Overall, 15,000 points were sampled sequentially for the four inputs over a fixed range [11]. The 17 outputs are obtained by solving the first principles-based model described in Section 3.1. Pre-processing of data typically involves several steps, such as data cleaning, data splitting, normalization, feature engineering, and sequence formation. The 15,000 data points for the inputs and output obtained from the model are consistent; hence, the data-cleaning process was not considered. The data is divided into 80% training, 10% validation, and 10% testing data sets.

To eliminate the scalability issues, the training data is normalized to a standard scale, such that all variables have the same level of importance and facilitate the training process. By appropriately adjusting the size of the input batch sequence (t), the model will be capable of capturing the temporal dependencies and patterns present in the historical data, facilitating more accurate and robust predictions from data-based models. For this study, the size of the input batch sequence 't' for each output is selected using a random search with the initial value of batch size as 15. The input sequences for training, testing, and validation have the following shapes: (11974, 15, 3), (1984, 15, 3), and (984, 15, 3).

Table 1 Traditional RNN, LSTM, and GRU model architectures

Model	Layer-1 units	Layer-2 units	Layer-3 units	Layer-4 unit	Total number of parameters
RNN	128	64	32	1	32385
LSTM	128	64	32	1	129441
GRU	128	64	32	1	97761

3.3 Dynamic data-based models – Recurrent Neural Networks

The dynamic data-based models under this study comprise the traditional RNN, LSTM, and GRU. In this study, identical architectural layouts were considered to maintain consistency across these models, as shown in Table 1. Based on the number of layers and units, the total number of parameters for the RNN, LSTM and GRU models is 32385, 129441, and 97761, respectively. Throughout the training process, the model parameters were updated using the ADAM (Adaptive Moment Estimation) optimizer, and each update was performed (batch size of 100 over 150 epochs). Apparently, the objective during the training process was to minimize the mean square error (MSE between the original and predicted output at time t) of the validation data. Moreover, the overfitting issue is also addressed by implementing regularization techniques and dropout techniques. The performance of the RNN models is evaluated based on the prediction MSE on the test data.

4. RESULTS & DISCUSSIONS

4.1 Solution of the three-phase dynamic model

Some of the output variables in the model are dependent not only on the inputs but also on other output variables, ultimately leading to implicit functions. For example, the gas holdup and pulp height are dependent on one another. These implicit differential equations are solved using the Scipy library in Python 3.8.17 by converting them into initial value problems. In Fig. 2, the profiles of four output variables are shown with respect to time and compared with those in the literature [10]. It is found that the profiles obtained are in accordance with those obtained by the authors in [10].

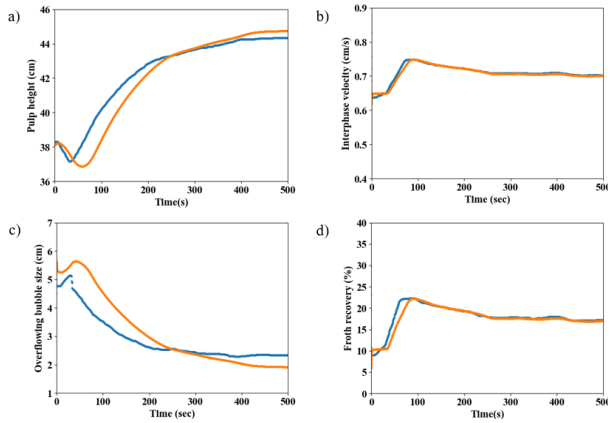


Fig. 2. Comparison of model results (blue) with literature (orange): a) pulp height vs. time; b) interphase velocity vs. time; c) overflowing bubble size vs. time; d) froth air recovery vs. time.

4.2 Data-based modeling and comparison study

Owing to their capabilities and broad applicability, deep learning models such as RNN, LSTM, and GRU are considered for modeling the above-described non-stationary data. Multi-Input Single Output Models were constructed in this regard for the considered system. **Figs. 3 – 5** present the training and validation MSE with respect to epochs for traditional RNN, LSTM, and GRU models, respectively, for some outputs such as pulp height, interphase gas velocity, interphase bubble size, and froth air recovery. In all the models, it is observed that the training and validation MSE decreases with epochs. The lack of a significant rise in validation MSE implies that the models are not exhibiting signs of overfitting.

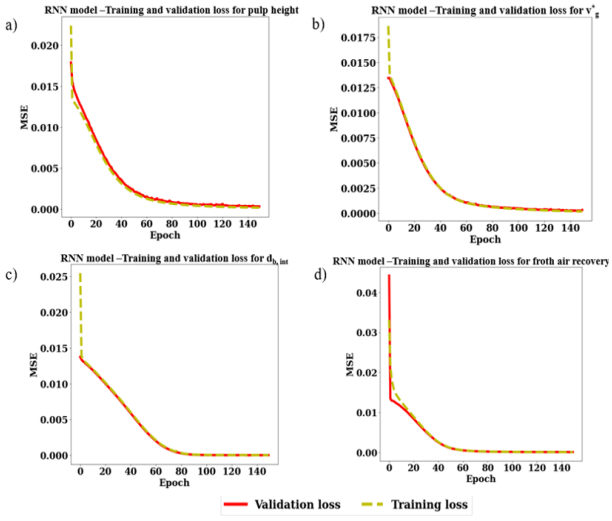


Fig. 3. MSE of the training and validation for the RNN model: a) Pulp height over epoch; b) Interphase gas velocity over epoch; c) Interphase bubble size over epoch; d) Froth air recovery over epoch.

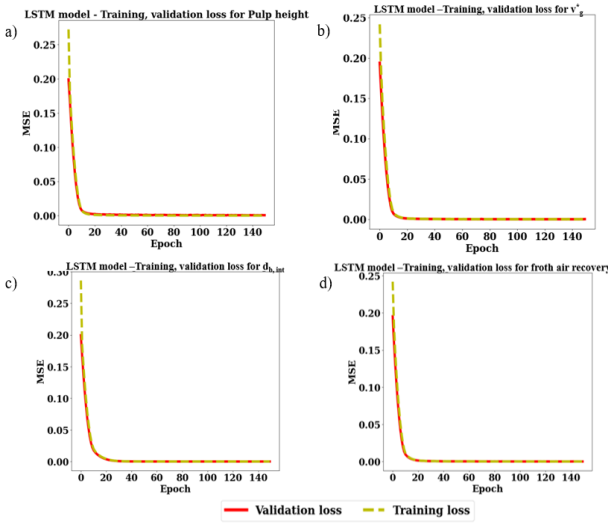


Fig. 4. MSE of the training and validation for the LSTM model: a) Pulp height over epoch; b) Interphase gas velocity over epoch; c) Interphase bubble size over epoch; d) Froth air recovery over epoch.

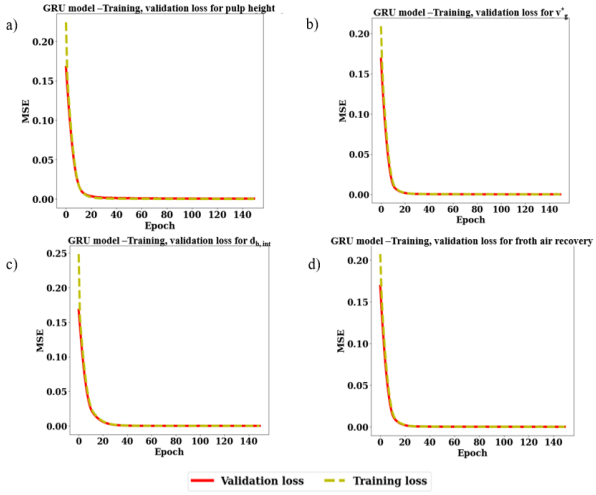


Fig. 5. MSE of the training and validation for the GRU model: a) Pulp height over epoch; b) Interphase gas velocity over epoch; c) Interphase bubble size over epoch; d) Froth air recovery over epoch.

In the case of the LSTM model, the MSE loss stabilizes and converges at an early stage, typically around 30 epochs **(Fig. 4)**, in comparison with RNNs (**Fig. 3**). Similarly, for the GRU model, both training and validation MSE decrease over the epochs and converge at an early stage, typically around 30 epochs (**Fig. 5**), compared to RNNs. This study indicates that the LSTM and GRU models learn and capture the underlying data patterns faster than the RNN model. **Fig. 6** shows the prediction of the test dataset by the traditional RNN, LSTM and GRU models for some outputs, namely, interphase bubble size, overflowing bubble size, concentrate flowrate, mass flowrate due to entrainment, mass flowrate due to true flotation, and mineral recovery.

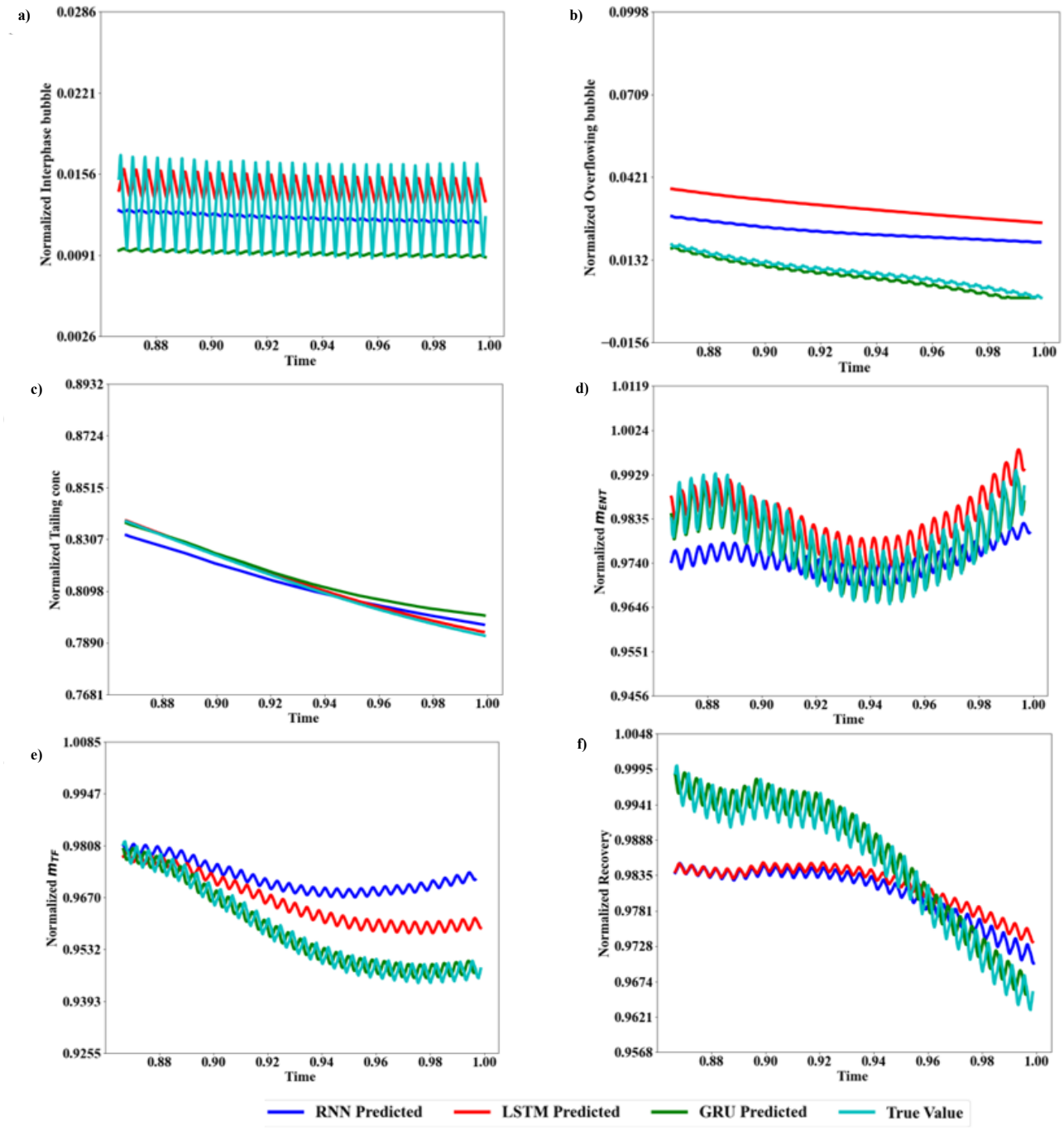


Fig. 6. Prediction of the test dataset by traditional RNN, LSTM, and GRU model: a) Interphase bubble size; b) Overflowing bubble size; c) Mineral concentration in tailing; d) Mass flowrate due to entrainment; e) Mass flowrate due to true flotation; f) Mineral recovery.

The predictions are shown in normalized scale. It was found that specific outputs such as interphase bubble size, mass flowrate due to entrainment, mass flowrate due to true flotation and mineral recovery exhibit notable fluctuations upon closer examination. On the other hand, outputs like gas holdup, pulp height, mineral concentration in pulp, and so on display comparatively negligible volatility with time. Nonetheless, as anticipated, the recurrent neural network models, particularly LSTM and GRU, exhibit adeptness in capturing fluctuation patterns through the utilization of input parameters and historical data.

Moreover, in comparison with the original/true profiles, it is found that the GRU model, in particular, demonstrates superior proficiency in capturing highly nonlinear output data such as interphase bubble size, mass flowrate due to entrainment, mass flowrate due to true flotation, and mineral recovery, outperforming both LSTM and RNN models. **Table 2** shows the comparison study of the

Table 2. Comparison study of traditional RNN, LSTM and GRU models for some of the outputs

Output	Model	Prediction MSE loss	Preferred model
Gas velocity	RNN	8.21e-07	RNN
	LSTM	1.10e-05	
	GRU	8.27e-07	
Interphase velocity	RNN	5.22e-05	GRU
	LSTM	1.49e-05	
	GRU	1.06e-05	
$d_{b,\ froth\ out}$	RNN	0.00020	GRU
	LSTM	0.00051	
	GRU	1.67e-06	
Froth air recovery	RNN	0.00010	GRU
	LSTM	1.08e-06	
	GRU	1.02e-05	
Q_{conc}	RNN	0.00070	GRU
	LSTM	0.00075	
	GRU	0.00048	
$C_{Tailing}$	RNN	8.85e-06	LSTM
	LSTM	1.08e-06	
	GRU	9.96e-05	
Mineral recovery	RNN	6.18e-05	GRU
	LSTM	6.26e-05	
	GRU	2.57e-06	

traditional RNN, LSTM, and GRU models for each output in terms of prediction MSE and the number of model parameters in the considered network. Based on these metrics and the prediction capability, the preferred model for each output is identified. The preferences for some of the outputs are presented in Table 2. Overall, in the majority, GRU models are preferable (for 9 out of 17 outputs) among the remaining. Overall, in the majority, GRU models are preferable (for 9 out of 17 outputs) among the remaining. This might be due to GRU's enhanced capability to model intricate dynamics within the data accurately.

5. CONCLUSION

A three-phase first principles-based model consisting of 61 DAEs was considered for the froth flotation process and solved for 17 variables. Owing to the computational complexity of this model, there was a need to develop data-based models for optimization and control studies of the flotation process. In this regard, 15,000 sequential data were sampled from the model with four inputs and 17 outputs. Subsequently, dynamic data-based modelling (multi-input single-output) was performed using deep learning algorithms such as RNN, LSTM, and GRU. The developed models were compared in terms of prediction MSE and parameter count. It was found that both GRU and LSTM models surpassed the traditional RNN, particularly for outputs prone to high fluctuations such as interphase bubble size. This study underscores the effectiveness of advanced neural network architectures in predictive modeling within mineral separation processes, offering profound insights and paving the way for enhanced optimization and control in industrial flotation operations.

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REFERENCES

[1] S. K. Kawatra and P. Darling, "Fundamental principles of froth flotation," *SME mining engineering handbook* 2, pp. 1517-1531, 2011.

[2] P. Quintanilla, S. J. Neethling and P. R. Brito-Parada, "Modelling for froth flotation control: A review," *Minerals Engineering*, vol. 162, p. 106718, 2021.

[3] M. Gharai and R. Venugopal, "Modeling of flotation process—An overview of different approaches," *Mineral Processing and Extractive Metallurgy Review,* vol. 37, no. 2, pp. 120-133, 2016.

[4] T. Imaizumi, "Kinetic consideration of froth flotation," *Proceedings of the Sixth International Mineral Processing Congress, 1963*. Pergamon Press, 1963.

[5] J. S. Guerrero-Pérez, and J.M. Barraza-Burgos, "A new mathematical model for coal flotation kinetics," *Dyna,* vol. 84, no. 203, pp. 143-149, 2017.

[6] E. C. Dowling, R. R. Klimpel and F. F. Aplan, "Model discrimination in the flotation of a porphyry copper ore," *Mining, Metallurgy & Exploration* 2, pp. 87-101, 1985.

[7] Y. Pu, A. Szmigiel, J. Chen and D. B. Apel, "FlotationNet: A hierarchical deep learning network for froth flotation recovery prediction," *Powder Technology*, vol. 375, pp. 317-326, 2020.

[8] A. C. Costa, F. V. Campos, L. R. Araujo, L. C. Torres, and A. P. Braga, "Deep architecture for silica forecasting of a real industrial froth flotation process," *Engineering Applications of Artificial Intelligence*, vol. 115, p. 105196, 2022.

[9] H. Zhang, Z. Tang, Y. Xie, J. Luo, Q. Chen, and W. Gui, "Grade prediction of zinc tailings using an encoder-decoder model in froth flotation," *Minerals Engineering*, vol. 172, p. 107173, 2021.

[10] P. Quintanilla, S. J. Neethling, D. Navia, and P. R. Brito-Parada, "A dynamic flotation model for predictive control incorporating froth physics. Part I: Model development," *Minerals Engineering*, vol. 173, p. 107192, 2021.

[11] P. Quintanilla, S. J. Neethling, D. Mesa, D. Navia, and P. R. Brito-Parada, "A dynamic flotation model for predictive control incorporating froth physics. Part II: Model calibration and validation," *Minerals Engineering*, vol. 173, p. 107190, 2021.