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Parameter Identification and Control of Integrating Time Delay Systems

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Abstract— **This paper presents the ideal relay-based modeling and control schemes for a class of time-invariant linear systems exhibiting integrating and time delay dynamics. Based on the important parameters in induced system response, a set of explicit equations is derived for evaluation of parameters associated with an assumed integrating plus time delay transfer function model. Under the adverse effects of sensor noise, a best-fit output is recovered by using a curve fitting method based on a Fourier series approach. The range of normalized dead time to which conditions for existence of limit cycles in an integrating and time delay systems is further enhanced by incorporating a feedback controller. For efficient and robust control design, a combination of feedforward proportional-integral (PI) and feedback proportional (P) controllers is utilized. Tuning guidelines for each of the controller gains are established through an integral of time-weighted error criterion. Illustrative examples are studied to validate the proposed algorithms for parameter identification and controller design. Furthermore, the efficacy of methods proposed is shown by using frequency and step response plots.**

I. INTRODUCTION

With the advent of modern control technology, investigations of robust methods for modeling and control of time delay systems have gained huge attention for enhancing the consistent performance and rate of manufacturing in process control and automation industries. In particular, many industrial systems, for instance, level control systems as reported in literature [1], are modeled in terms of integrating and time delay dynamics; therefore, study of such systems using the closed loop configuration in the continuous-time domain is required for system modeling and parameter identification of unknown dynamics in terms of equivalent transfer function models using various identification tools [2], [3]. During the late 80s, identification tests based on the relay feedback approach were one of the best-adapted techniques in various industries due to their features of simplicity, automatization, and versatility, where the output waveform can easily be controlled by changing the relay settings. One of the key roles of system (or process) identification based on relay feedback lies in the autotuning of controllers, where the tuning guidelines for each unknown controller gain are designed by using estimated model parameters and user-specified criteria.

Considering the two positions of relay (on/off), the necessary and sufficient conditions for limit cycle existence under a squared waveform at relay output using the Fourier series and time domain analyses suggested in [4], [5]. Furthermore, analyses proposed by them have been carefully studied and attempted in [6] for investigations of limit cycle oscillations with their existence, uniqueness, and local stability. In the beginning, the existence and stability conditions for limit cycle oscillations are developed in [7] using the describing function (DF) approach, which considers the involvement of balance among harmonic components. In the last two decades, the conditions to improve the region for the existence of sustained oscillations followed by global stability are discussed by using Poincaré maps in [8]. During the mid-80s, when the autotuners based on relay feedback were invented for bringing ease in designing tuning rules for classical controllers as described in [9], its exploration towards the system modeling and parameter identification of various industrial processes started. Owing to the presence of erroneous sensors at output, it may fail in obtaining the actual measurements that may cause numerous switching of the relay and hence results in different values of the ultimate period of plant dynamics; therefore, a hysteresis band is suggested by authors for minimization of measurement noise issues. Luyben [10] has implemented a technique of autotune variation (ATV) to study the distillation columns. To estimate the crucial points on the Nyquist curve, Shen *et al.* [11] proposed an approach based on a dual-input describing function (DIDF) where a biased value is selected to overcome the measurement noise impact on model parameter estimates, and such analysis was especially applicable for stable systems. Liu and Gao [3] have proposed robust algorithms for system parameter identification using step response analysis with industrial applications. Many authors proposed methodologies (see [12] and references therein) for enhanced identification of industrial processes using different configurations of relay and step response-based identification tests with a limitation of computation of every system model parameter. Panda *et al.*[1] discussed the limit cycle analysis for the identification of systems exhibiting different integrating systems with/without non-minimum phase and time delay dynamics. Bajarangbali *et al.* [13], Ghorai *et al.* [14], Pandey and Majhi [15] have derived mathematical expressions for identification of stable, unstable, and integrating systems using frequency and time domain methods. When the system output is contaminated with measurement noise, a method that minimizes the multiple relay switching is proposed in [16], however, causing an increment in the overall order of process dynamics. Pandey and Majhi [17] have proposed an analytical set of expressions for off-line/on-line relay-based identification algorithms for stable time delay systems

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followed by experimental validation on coupled tank systems without any discussion related to integrating systems. For efficient design of model-based controller tuning rules, various approaches have been reported on the basis of internal model control schemes in [18], [19], [20] and selected sensitive range against step load disturbance with improved performance, which can be inferred from [21] and references therein.

Therefore, the major contributions of the paper are summarized below:

- It is worth mentioning that a set of explicit and simple expressions is derived using the collected information from the sustained oscillatory responses and relay settings as compared to the reported methods where a nonlinear solver is required to obtain parameters from a group of nonlinear mathematical equations.
- As the off-line identification method may fail in guaranteeing the accuracy of the model parameters because of the approximated gains of the relay, a proportional controller is connected in an inner feedback loop, which may stabilize the dynamics of overall integrating systems.
- Under the adverse effects of input-output disturbances, a proportional-integral controller is connected in a feedforward path by an appropriate design of controller gains using balanced tuning rules, followed by the implementation of a curve fitting technique based on Fourier series analysis for reconstruction of the original limit cycle output.

The paper is arranged as follows: The proposed identification and control schemes are described in Section II, followed by derivations and sensor noise issues in system identification in Section III. Simulations of typical examples are included in Section IV. The conclusions appear in Section V.

II. Proposed Identification and Control Schemes

This section deals with the discussion and development of identification and control schemes for continuous-time systems exhibiting integrating and time delay dynamics. Figs. 1 and 2 show a block diagram of off-line and on-line identification tests under relay control with or without feedforward and feedback controllers. Initially, an identification test is to be conducted with a relay with asymmetrical amplitudes, which may assist in eliminating the effects of static load disturbances by varying its relay settings. Based on the important information from the yielded system output in terms of limit cycle responses around the setpoint, attempts are made to develop an explicit set of expressions for

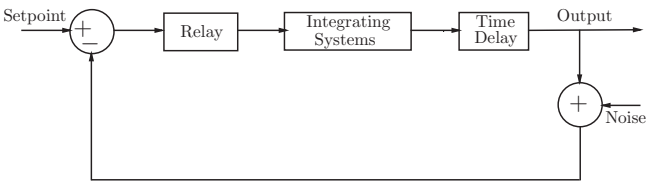


Fig. 1. Relay-based off-line identification scheme

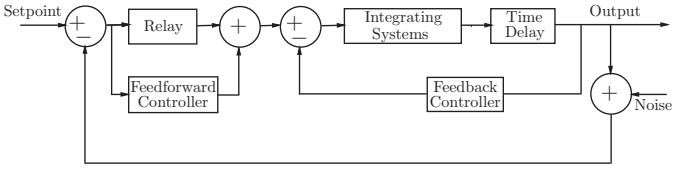


Fig. 2. Relay-based on-line identification and control schemes

frequency domain modeling of an unknown system transfer function model. Prior to derive the expressions for each unknown associated with the transfer function model, the expression for non-linear device (relay) as an identification tool is obtained (see [22] for complete derivation).

Assuming the input to an ideal relay is to be sinusoidal

$$e(\phi) = A_m sin(\phi) \tag{1}$$

where A_m denotes the mean of the ultimate amplitudes and ϕ represents the oscillation frequency of an input signal.

Based on input-output information, the characteristics of a relay with asymmetrical amplitudes can be mathematically expressed as

$$u(\phi) = \begin{cases} h_1 & \text{for} \quad 0 < \phi < \pi + \phi_1 \\ -h_2 & \text{for} \quad \pi + \phi_1 < \phi < 2\pi - \phi_1 \\ h_1 & \text{for} \quad 2\pi - \phi_1 < \phi < 2\pi \end{cases} \tag{2}$$

where $\phi_1 = \sin^{-1}(A_m^{-1} B_m)$.

Using the describing function approach as detailed in [22], the equivalent expression for ideal relay gain with asymmetrical amplitudes is derived by neglecting the higher order frequency terms because the system model depicts low-pass filter characteristics. The asymmetrical relay gain expression in simplified form is written as

$$N(A_m, B_m) = \frac{2(h_1 + h_2)}{\pi A_m} \sqrt{1 - \left(\frac{B_m}{A_m}\right)^2} = N(\cdot) \tag{3}$$

where $N(A_m, B_m)$ or $N(\cdot)$ signifies the equivalent asymmetrical relay gain, and A_m denotes an average value of ultimate amplitudes in the above (A_p) and below (A_v) sections of the waveform with respect to setpoint as

$$A_m = \frac{A_p + A_v}{2} \tag{4}$$

and a bias B_m in amplitude is represented as

$$B_m = \frac{|A_p - A_v|}{2} \tag{5}$$

Assumption 1: The assumed structure of system models is to be known.

Assumption 2: The system transfer function models are considered to exhibit linear time-invariant characteristics.

Assumption 3: Time delay (θ) is a crucial parameter to be evaluated with the system model parameter.

III. Development of Explicit Equations

From [9], the limit cycle existence condition for a closed loop system is written in terms of loop gain as

$$N(A_m, B_m) G_m(j\omega) = -1 \tag{6}$$

A. Relay-Based Off-line Identification Test

A general transfer function model is considered in Fig. 1 to represent integrating systems with time delay as

$$G_m(s) = \frac{ke^{-\theta s}}{s} \tag{7}$$

Using the frequency domain representation, the transfer function $G_m(j\omega)$ for integrating systems with time delay is expressed as

$$G_m(j\omega) = \frac{ke^{-j\omega\theta}}{j\omega} \tag{8}$$

Substituting the governing expressions of $N(A_m, B_m)$ and $G_m(j\omega)$ from (3) and (8) in (6), we get

$$\frac{2(h_1+h_2)}{\pi A_m}\sqrt{1-\left(\frac{B_m}{A_m}\right)^2}\left(\frac{ke^{-j\omega\theta}}{j\omega}\right) = -1 \tag{9}$$

where $\omega = \dfrac{2\pi}{T_p}$.

The governing expressions as obtained from gain and phase conditions are expressed as

$$\frac{2(h_1+h_2)}{\pi A_m}\sqrt{1-\left(\frac{B_m}{A_m}\right)^2}\left(\frac{k}{\omega}\right) = 1 \tag{10}$$

$$-\theta\omega - \frac{\pi}{2} = -\pi \tag{11}$$

The expressions of steady state gain and time delay from (10) and (11) are derived as

$$k = \left[\frac{\pi^2 A_m}{T_p(h_1+h_2)}\left(1-\left(\frac{B_m}{A_m}\right)^2\right)^{-1/2}\right] \tag{12}$$

$$\theta = \frac{T_p}{4} \tag{13}$$

B. Relay-Based On-line Identification Test

During the on-line identification test, a feedback controller in an inner feedback loop is connected to bring stabilization in overall system dynamics and to enhance the range for the existence of sustained oscillations with respect to different values of time delay. The modified transfer function model due to an inclusion of an inner loop controller as shown in Fig. 2 becomes

$$\tilde{G}_m(s) = \frac{ke^{-\theta s}}{(s+kk_fe^{-\theta s})} \tag{14}$$

Assuming the smaller value of time delay as incurred from signal propagation, the expression using the Taylor's series approximation is expressed as

$$e^{-\theta s} \approx (1-\theta s) \tag{15}$$

Incorporating the approximation in (14), the modified transfer function is represented as

$$\tilde{G}_m(s) = \frac{ke^{-\theta s}}{(1-kk_f\theta)s+kk_f} \tag{16}$$

Subsequently, the stabilized transfer function model is written as

$$\tilde{G}_m(j\omega) = \frac{\tilde{k}e^{-j\omega\theta}}{(j\omega\tilde{\tau}+1)} \tag{17}$$

where $\tilde{k} = \dfrac{1}{k_f}$ and $\tilde{\tau} = \dfrac{1-kk_f\theta}{kk_f}$.

Substituting $N(A_m, B_m)$ and $\tilde{G}_m(j\omega)$ from (3) and (17) in (6) results

$$\frac{2(h_1+h_2)}{\pi A_m}\sqrt{1-\left(\frac{B_m}{A_m}\right)^2}\frac{\tilde{k}e^{-j\omega\theta}}{(j\omega\tilde{\tau}+1)} = -1 \tag{18}$$

Using the gain and phase conditions, the expressions are derived as

$$\frac{2(h_1+h_2)\tilde{k}}{\pi A_m\sqrt{(\omega\tilde{\tau})^2+1}}\sqrt{1-\left(\frac{B_m}{A_m}\right)^2} = 1 \tag{19}$$

$$-\omega\theta - \tan^{-1}(\omega\tilde{\tau}) = -\pi \tag{20}$$

Evaluating the expression of $\tilde{\tau}$ using the gain condition as

$$\tilde{\tau} = \frac{T_p}{2\pi}\left[\left(\frac{2(h_1+h_2)\tilde{k}}{\pi A_m}\sqrt{1-\left(\frac{B}{A_m}\right)^2}\right)^2 - 1\right]^{1/2} \tag{21}$$

Simplifying the phase condition, the expression for θ can be written as

$$\theta = \frac{T_p}{2\pi}\left[\pi - \tan^{-1}\left(\frac{2\pi\tilde{\tau}}{T_p}\right)\right] \tag{22}$$

The remaining unknown parameter ($\tilde{k}$) is computed by taking the ratios among the area under system output waveform to system input waveform over a complete oscillation period as

$$\tilde{k} = \frac{\int_0^{T_p} y(t)dt}{\int_0^{T_p} u(t)dt} \tag{23}$$

C. Tuning Guidelines for Model-Based Controllers

This section presents the tuning guidelines for model-based design of feedforward and feedback controllers for integrating systems with time delay. Note that there is always a requirement of a robust controller, which may make the system less sensitive to the effects of input and output disturbances. In this context, the presented work mainly focuses on the minimal amount of energy spent by the controller in ascertaining the improved transient performance with respect to the existing methods. In the literature, a large gain value of a proportional controller is considered to achieve better transient performance, resulting in the response exhibiting poor damping. Such a control strategy may cause a substantial amount of faults in actuators and hence deteriorate the overall assessment of the system.

Motivated from the works reported in [19], tuning rules based on the integral of time-weighted error criterion where a balance among the proportional gain and integral gain of the controller is targeted to keep fictitious control action to be

less aggressive. For integrating systems with time delay, the proportional controller is connected in a feedback loop for stabilizing the overall system dynamics, and a proportional-integral controller is connected in series to the modified system dynamics for elimination of incurred static load disturbance at the system input.

1) Feedback Controller Design: In order to stabilize the overall system dynamics by reallocating the poles of the original system, a range of feedback controller gain is obtained from (16) as

$$0 < k_f < \frac{1}{k\theta} \tag{24}$$

2) Feedforward Controller Design: For improving the output transient performance of the integrating system model and robustness towards the input disturbances, the feedforward controller is designed. The expression of a PI controller output in terms of velocity form is represented by

$$\dot{u}(t) = k_p \dot{e}(t) + k_i e(t) \tag{25}$$

When the system output approaches to a steady state, $\dot{u}(t) = 0$, therefore, integration on both sides of time-weighted absolute error and absolute derivative of error with time-weighted is expressed as

$$\frac{k_p}{k_i} \underbrace{\int_0^\infty t|\dot{e}(t)|dt}_{\text{ITADE}} = \underbrace{\int_0^\infty t|e(t)|dt}_{\text{ITAE}} \tag{26}$$

From the closed loop error dynamics, the expression for step response is written as

$$e(t) = \begin{cases} \tilde{k} & \text{if } 0 \leq t < \theta \\ \tilde{k}\exp\left(-\frac{t-\theta}{\tilde{\tau}}\right) & \text{if } t \geq \theta \end{cases} \tag{27}$$

Substituting the closed loop error expressions from (27) in the performance indices as detailed in (26), the proportional and integral controller gains are derived as

$$k_p = k_i \tilde{\tau} \left(\frac{0.5\theta_n^2 + \theta_n + 1}{\theta_n + 1} \right) \tag{28}$$

where $\theta_n = \theta/\tilde{\tau}$ signifies the time delay in normalized form. In addition, the expression of controller integral gain k_i is determined by using the index of sluggishness [20] by considering the relationship between open loop and closed loop control response time measures. Furthermore, the mean residence period in an open loop system is obtained as

$$\frac{\dot{\tilde{G}}_m(0)}{\tilde{G}_m(0)} = -(\theta + \tilde{\tau}) \tag{29}$$

The fictitious transfer function model for closed loop system $\hat{G}_m(s)$ is obtained as

$$\hat{G}_m(s) = \frac{\tilde{k}e^{-\theta s}(k_p s + k_i)}{s(\tilde{\tau}s + 1) + \tilde{k}e^{-\theta s}(k_p s + k_i)} \tag{30}$$

On the same lines to the inner loop system $\tilde{G}_m(s)$, the mean residence period in the closed loop system is evaluated as

$$\frac{\dot{\hat{G}}_m(0)}{\hat{G}_m(0)} = -\frac{1}{\tilde{k}k_i} \tag{31}$$

Finally, the mean residence period as obtained for open loop and closed loop systems is equated together, which assists in establishing the expression for integral gain k_i as

$$k_i = \frac{1}{\tilde{k}\tilde{\tau}(\theta_n + 1)} \tag{32}$$

D. Existence of Sustained Oscillatory Responses

As the relay-based identification experiment is commenced for parameter estimation of time delay systems with integrating dynamics that cannot be accomplished until a sustained oscillatory response at system output is ensured. Prior to establishing the explicit set of analytical expressions, the limit cycle existence condition for such systems has to be guaranteed. In literature, the contributions in regards to the investigation of sustained oscillations in relay feedback systems are primarily addressed by [4]. Assuming two positions (on/off) in relay operation result in a squared waveform at relay output, the limiting conditions for the existence of sustained oscillations are derived by using Fourier series analysis. In the early 1940s, the existence conditions were explored by utilizing the concept of DF analysis, where a method of balancing the harmonics was considered and limited for particular systems. Since then, existence and uniqueness conditions together with stability analysis have been investigated by leveraging the Poincaré maps in [8].

In practical applications, the dynamics of various systems associated with industries show time delay and nonlinear characteristics, which may be expressed in terms of a time delay linear model through a linearization approach. Time delay is ubiquitous, and its role is crucial in the generation of sustained oscillatory responses because the overall order of system dynamics can be increased by incorporating the infinite number of poles. As the system order is raised, then its response in the frequency domain approaches the third quadrant with an intersection with the imaginary axis, ensuring the generation of a limit cycle around the setpoint. In contrast, linear time-invariant systems without the time delay effect experience difficulty in crossing the third quadrant and hence fail to bring the sustained oscillations that can be inferred from Fig. 3. Note that the assumed system

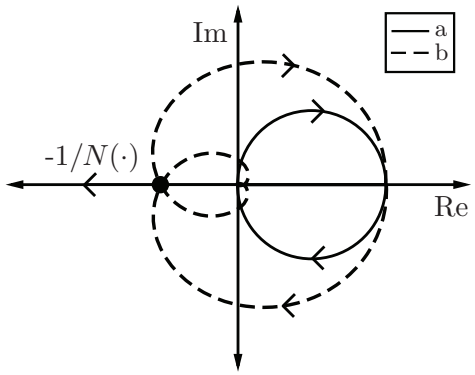


Fig. 3. Frequency response plots for system models (a) without time delay, (b) with time delay

with time delay dynamics under relay feedback control

exhibits sustained oscillatory responses when its Nyquist plot possesses a solution with the negative real axis as

$$G_m(j\omega) = -\frac{1}{N(A_m, B_m)} \tag{33}$$

Under the application of feedforward and feedback controllers, the range for existence is further enhanced, which indirectly depends on the gain of the inner loop feedback controller as

$$G_m(j\omega) = -\frac{1}{N(A_m, B_m) + G_{c_1}(j\omega) + G_{c_2}(j\omega)} \tag{34}$$

E. Effects of Measurement Noise

In process control and automation industries, system output is quite sensitive to output disturbances because of the presence of inaccurate sensors connected at the output, leading to the erroneous evaluation of unknown system model parameters. Under the noisy environment, accurate estimation of unknown parameters based on the derived mathematical set of expressions may not be guaranteed. Measurements made from the sustained oscillatory response can be recovered by using the well-established signal processing techniques. In this draft, a noise-free system output is recovered by using the curve fitting approach based on Fourier series analysis, where a best-fit is selected from a waveform oscillating with principal harmonic components. Subsequently, the measured quantities from a reconstructed output $\hat{y}(t)$ are supplied in the derived set of explicit expressions for estimating the unknown parameters of assumed system models. To show the robustness of the proposed algorithm against noisy environments, a white Gaussian noise with a mean of zero and a variance of non-zero amplitude is added at the output. The governing equation through which a best-fit for noisy limit cycle is retrieved by considering the following equation as

$$\hat{y}(t) = p_0 + \sum_{i=1}^{N} [p_i \cos(i\omega t) + q_i \sin(i\omega t)] \tag{35}$$

where $\hat{y}(t)$, ω the system output and oscillation frequency are to be evaluated from a best-fit output.

The important coefficients (p_0, p_i, q_i) $\forall i \geq 1$ of Fourier series are represented as

$$\begin{aligned} p_0 &= \frac{1}{2T_p}\int_{-T_p}^{T_p} \hat{y}(t)d(\omega t) \\ p_i &= \frac{1}{T_p}\int_{-T_p}^{T_p} \hat{y}(t)\cos(i\omega t)d(\omega t) \\ q_i &= \frac{1}{T_p}\int_{-T_p}^{T_p} \hat{y}(t)\sin(i\omega t)d(\omega t) \end{aligned} \tag{36}$$

IV. Illustrative Examples

Two examples from literature are included to validate the proposed identification and control algorithms. An ideal relay is considered to excite the system dynamics while generating the sustained oscillatory responses around the reference input. The amount of parametric identification error among estimated and true system parameters is calculated by using an integral of absolute error (IAE) index as $\text{IAE} = \int_0^{\omega} |(G_m(j\omega) - G(j\omega))/G(j\omega)|d\omega$ where $G_m(j\omega)$ is the identified system model, $G(j\omega)$ is true system, and ω is the oscillation frequency of system, respectively.

A. Example 1

Now assume a pure integrating plus time delay system studied in [18] as $G(s) = \frac{e^{-s}}{s}$. Under the asymmetrical relay feedback experiment, the above system yields the information from sustained oscillatory response as $A_m = 1.0995$, $B_m = 0.0997$, and $T_p = 4.0328$, respectively. The important parameters A_m, B_m, and T_p are substituted in the mathematical expressions (12) and (13) as derived in Section III to obtain the unknown parameters during off-line identification as $k = 1.2281$ and $\theta = 1.0082$ with the parametric error in terms of IAE as 0.3585. In a practical environment, a chattering effect is commonly observed in system output due to faults in sensors that may lead to numerous switching at the relay. To show the robustness of the identification scheme proposed in this paper, a white Gaussian noise at system output of 0.1 NSR (noise to signal ratio) is added. Subsequently, a noise-free limit cycle output is retrieved by using the curve fitting method, and the identified system model under sensor noise is recovered as $k = 1.2699$ and $\theta = 1.0402$ resulting in the parametric error index IAE as 0.4250. The comparison between identified system transfer function models with/without noise and true systems is compared through the plots as shown in Fig. 4 based on frequency response. Under the application of

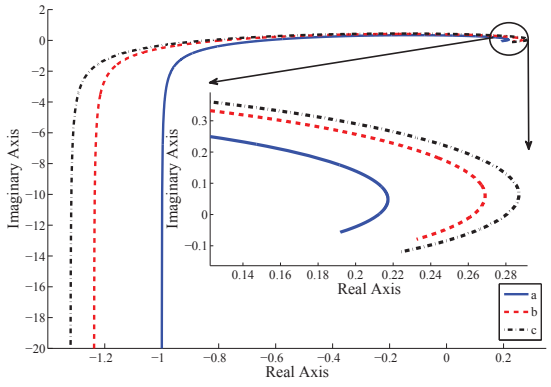


Fig. 4. Frequency response plots for Example 1: (a) true system, (b) proposed model without noise, (c) proposed model with noise

relay-based on-line identification tests, the initial gains of controllers are chosen as per the guidelines suggested in the Ziegler-Nichols method, resulting in the model parameter without measurement noise as $k = 1.0061$ and $\theta = 0.9992$ with IAE $= 0.0095$ and with noise as $k = 1.0524$ and $\theta = 0.9882$ with IAE $= 0.0833$. To achieve the improved transient performance, tuning guidelines for controllers in feedforward and feedback paths are obtained through the derived set of analytical expressions (24), (28), and (32), respectively.

B. Example 2

Consider the integrating and time delay system dynamics $G(s) = \dfrac{e^{-s}}{5s}$ as studied in [2]. Under the asymmetrical relay test, sustained asymmetrical output around the reference input is obtained by application of non-zero relay settings as $h_1 = 1$ and $h_2 = -1.2$. During off-line identification, the model parameters are obtained after substituting the important information $A_m = 0.2202$, $B_m = 0.0200$, and $T_p = 4.0370$ in (12) and (13) as $k = 0.2459$ and $\theta = 1.0092$ with IAE $= 0.3576$. Under the noisy environment, the robustness of the algorithm proposed is verified by an addition of non-zero variances to obtain the sustained oscillations with 10 dB, 15 dB, and 20 dB SNR (signal to noise ratio), respectively. A noise-free output is obtained by using the waveform fitting method, and information regained from the limit cycle is supplied in a mathematical set of derived expressions. During the on-line identification test, the measured quantities are substituted in governing expressions (21), (22), and (23) to obtain the system model parameters as $k = 0.2001$ and $\theta = 0.9999$ with IAE $= 0.0054$. From the identified system models, the controllers in feedforward and feedback paths are designed from (24), (28), and (32) as $k_f = 2.2361$, $k_p = 1.4597$, and $k_i = 1.00006$ to get the enhanced transient assessment as shown in Fig. 5, where $\pm 2\%$ perturbations in identified model parameters are considered to show the effectiveness of the proposed controller against variations of system parameters.

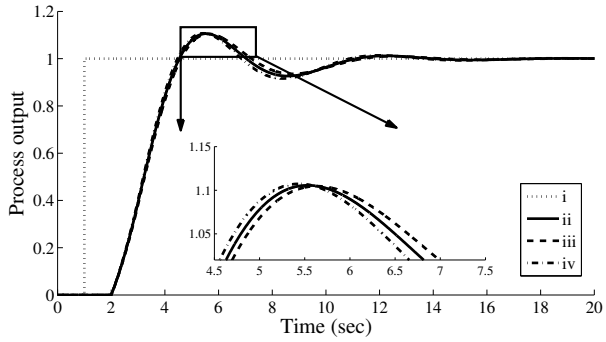


Fig. 5. Step response plots for Example 2: (i) reference input, (ii) proposed controller, (iii) proposed controller with 2% model parameter perturbation, (iv) proposed controller with -2% model parameter perturbation

V. CONCLUSIONS

In this paper, off-line and on-line identification schemes followed by the development of expressions under a single relay feedback test for parameter estimation of integrating systems with time delay are presented. Considering the measured quantities from the faulty sensor at the system output to be inaccurate, a clean limit cycle is reconstructed by using a method of curve fitting based on the Fourier series approach. From the recovered data, the measured quantities are substituted in the derived set of mathematical expressions to retrieve the system model parameters under off-line and on-line conditions against the measurement noise. Using the concept of balanced tuning rules, gains of controllers connected in feedforward and feedback paths are derived to achieve better transient performance. The merit of the controller proposed in this monograph is illustrated by considering the perturbations in the identified model parameters through step response plots.

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