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UDE based Robust Predictive Control for CoM Tracking of Humanoid Robot

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***Abstract*— This work proposes a new robust walking control design for humanoid robot. Using a representative three-dimensional linear inverted pendulum mode (3D-LIPM) of a humanoid, a Center of Mass (CoM) tracking controller is designed using Continuous Time Predictive Control (CTPC) approach. To achieve robustness, the controller is augmented by estimate of the disturbance obtained using Uncertainty and Disturbance Estimator (UDE). Closed loop stability of the overall system is established. Next, performance of the UDE based robust Continuous Time Predictive Control (UDE+CTPC) for CoM trajectory tracking in presence of external disturbances is demonstrated through simulations.**

I. INTRODUCTION

Robust walking control of humanoid robots has been an area of active research due to various complexities like coupled dynamics, inherent instability, and presence of external disturbances [1]. Several simplified models have been proposed for design of the balance and walking control algorithms wherein the inverted pendulum model (IP) represents one such widely used reduced order model for these robots [2]. In [3], IP model was modified by restricting the CoM height in horizontal plane, which results in a LIPM model. In [4], a $3-$D representation of LIPM model has been proposed and used for generation of walking patterns. Using 3D-LIPM model, a preview control for controlling Zero Moment Point (ZMP) for stabilization of the biped is studied in [5] whereas a ZMP tracking control using improved preview control for stabilization has been proposed in [6].

As interaction with the environment forces needs to be controlled to stabilize the robot, stabilization of the robot using ankle torque as well as ZMP control by CoM acceleration has been studied in [6]. Push-recovery in legged robots is a prime concern as it adversely affects the stability of robot. In such scenarios external disturbances acting on the robot can degrade the overall performance and therefore the issues need to be addressed. To this end, a stabilizing controller based on capture point (CP) is proposed in [7] wherein the LIP model is used to systematically find out the capture point as well as to design the CP tracking contoller. Next, in [8], the authors have proposed novel concept of N-step capturability and a stabilizing control based on the theory. In [9], gaits have been adapted considering the disturbance acting on the robot to generate the CoM trajectory. Various gaits based on mathematical model, stability criteria based gaits and gait generation based on learning algorithm have been studied in [10]. In [11] authors proposed a hybrid linear inverted pendulum (H-LIP) based apriodic walking patterns. A ZMP/CoM walking pattern based on rolling sphere model and control design comprising of ZMP as well as CoM controller ensuring robustness against disturbances caused by contact forces/moments is proposed in [12]. Design of a sliding mode control for controlling the joint angles can be found in [13] wherein authors achieved CoM tracking by controlling the joint angles.

Design of the walking controllers for humanoid robots affected by plant uncertainties and external disturbances is posed with critical challenges. Many robust control laws have appeared in the literature. Stable walking of humanoid using ESO based uncertainty estimation and robust control of a $3-$D LIPM has been studied in [14]. A robust CoM tracking controller for LIPM has been proposed in [15]. In [16], a higher order ESO for robustifying a joint servo tracking controller of a humanoid is proposed. The proposed controller offers superior performance than the controller based on a high order sliding mode observer. It is important to note that [16] has not robustified the outer balance and walking controller which need to be addressed.

Continuous Time Predictive Control (CTPC) proposed by [17], [18] is a systematic method to design the controller. Here, system output is predicted through Taylor's series expansion. Quadratic performance index based on the prediction error and control effort is minimized to obtain an optimal controller. Various applications of CTPC such as in robotics [19] and design of guidance law [20] have appeared in the literature. The design of predictive control needs exact mathematical model of the system being controlled. Hence the designed controller may offer degraded performance due to model uncertainties and external disturbances. To address this issue, the predictive controller has been robustified using various approaches. For example, in [21], [22] predictive control has been robustified by using Extended State Observer (ESO) estimated states and uncertainties. Application of ESO based robust control for constrained MIMO system has been reported in [23] whereas in [24], predicted state vector has been modified based on the linear proportional-integral-differential error to achieve the robustification.

Performance of the CTPC control law can be restored by estimating the effects of the disturbance and cancelling it by augmenting the control with the estimate. Various disturbance estimation techniques have appeared in literature

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for design of the robust control laws. For example, [25], [26] proposed a extended state observer (ESO) for estimating the uncertainties and system states in an integrated manner. Similarly, a sliding mode disturbance observer for estimating the disturbances and cancelling the same in controller is presented in [27]. A time-delay control (TDC) approach has been presented in [28] wherein the effects of parametric uncertainty and disturbances are estimated by using past information of the plant states and control. Progressing the line of thinking of TDC further while addressing some of its issues, [29] proposed an Uncertainty and Disturbance Estimator (UDE), where the effects of the disturbances are estimated by employing an appropriately designed filter. Performance improvement of UDE based robust controllers with $\alpha-$filter has been studied in [30]. Detailed analysis of $\alpha-$filter design and robust control with finite control bandwidth can be found in [31]. Robustification of contollers using UDE for various applications have been reported widely in the literature. In [32], UDE based robust feedback linearization control as well as state observer was designed for control of the robot manipulator. Robust consensus of non-linear multi agents with $\alpha-$ filter based disturbance estimate is investigated in [33]. Issue of control of uncertain nonlinear systems with state delays has been analyzed in [34] whereas UDE based robust backstepping controller for quadrotor trajectory tracking in presence of uncertainties as well as disturbances has been studied in [35].

Following the same line of thinking of disturbance estimation and rejection approach, a UDE based robust and optimal continuous-time predictive control for tracking CoM of a humanoid robot is proposed in this work. The important features of the proposed scheme are, (i) a continuous time predictive control is robustified using a UDE approach, (ii) proposed approach does not need knowledge of the disturbance model and, (iii) the approach is extended to design the outer CoM tracking loop responsible for balance and walking of humanoid robot, unlike other reported work where robust control is designed for joint servo loop. Closed loop stability of the $3-$D LIPM representing the simplified humanoid robot model along-with the UDE+CTPC is established and simulation results are presented to demonstrate efficacy of the proposed design.

The remaining paper is organized as follows. Section II describes the $3-$D LIPM model of the *Mahru-I* humanoid as well as the design of a output tracking predictive controller for the same. The UDE based robust predictive control is presented in Section III, closed-loop stability analysis is presented in Section IV. Section V presents the simulation results, and Section VI concludes this work.

II. PROBLEM FORMULATION

Legged robot walking is classified as a static and dynamic walking. In static walking the projection of the robot CoM never leaves the support polygon whereas in dynamic walking the projection of the robot CoM leaves the support polygon for a small period [6]. Therefore, during dynamic walking, CoM need to be controlled precisely failing which robot may topple down. Walking based on CoM pattern [36], additive ZMP and CoM controller based posture and walking controller [12] and ZMP and CoM feedback based preview controller for stable dynamic walking [5], are some results to cite. In this paper the robust predictive control design for the *Mahru-I* robot's reduced order model is proposed. The control objective is to design robust predictive control such that the CoM of the LIPM tracks the reference CoM satisfactorily while external disturbance acting on the robot.

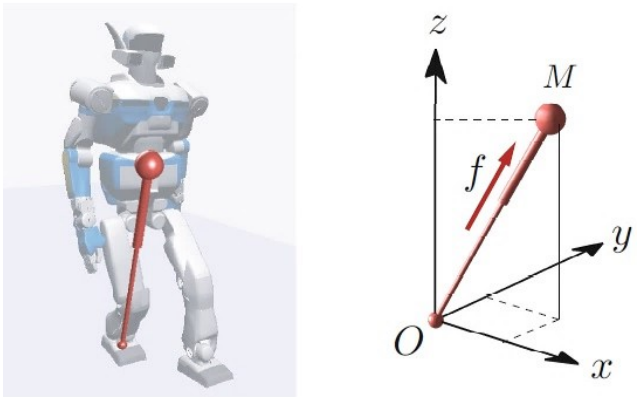


Fig. 1. $3-$D LIP representation of a humanoid robot [6]

A. Mathematical Model

For the design, a humanoid robot walking in 3D space is represented by an imaginary 3D-LIPM shown in Fig. 1. The LIPM rotates freely about its pivot O, which resembles the ankle of the support leg during single support phase of walk. Here robot CoM is at the other end of the mass-less prismatic joint [6]. Now, consider constraint plane $z = k_x x + k_y y + z_c$ where k_x and k_y are the slopes and z_c is the height of the constraint plane. The dynamics of the $3-$D LIPM can be obtained as [4], [12]

$$\ddot{x} = \frac{g}{z_c}x + \frac{1}{mz_c}\tau_y \quad (1)$$

$$\ddot{y} = \frac{g}{z_c}y - \frac{1}{mz_c}\tau_x \quad (2)$$

where m is the mass of the pendulum, g is gravitational acceleration, (x, y) are the CoM displacement coordinates and τ_x and τ_y are the moments about the x and y-axis respectively. Defining the states as $q_x = x$ and $q_y = y$ i.e. with generalised states $q_i, \;\; i = x, y$, the dynamics of (1)-(2) is written in generalised states as

$$\ddot{q}_i = aq_i + b_o\tau_i \quad (3)$$

$$Y_i = q_i \quad i = x, y \quad (4)$$

where Y_i is the output, $a = \frac{g}{z_c}$, and $b_o = \pm\frac{1}{mz_c}$. From (3)-(4), it can be seen that the relative degree is 2 for both the predicted outputs.

B. Continuous Time Predictive Control (CTPC)

Following the output tracking formulation of the CTPC [17], [18] and noting that the relative degree for both outputs is 2, the output at time $(t+\delta_i)$ due to the present control $\tau_i(t)$ is predicted by approximating $q_i(t+\delta_i)\ \ i=x,y$ by a second order Taylor's series at t as

$$Y_i(t+\delta_i) \approx q_i(t) + \delta_i \dot{q}_i(t) + a\frac{\delta_i^2}{2}q_i(t) + b_o\frac{\delta_i^2}{2}\tau_i(t) \quad (5)$$

where $\delta_i > 0$ is a small time interval defined as a prediction horizon [17] at which the output q_i is predicted. Similarly, the reference output is expanded in second order Taylor's series as

$$Y_i^\star(t+\delta_i) = q_i^\star(t) + \delta_i \dot{q}_i^\star(t) + \frac{\delta_i^2}{2}\ddot{q}_i^\star(t) \quad (6)$$

Defining the predicted output tracking error at time $(t+\delta_i)$ as $e_i(t+\delta_i) = Y_i(t+\delta_i) - Y_i^\star(t+\delta_i)$, one gets

$$\begin{aligned} e_i(t+\delta_i) &= e_i(t) + \delta_i \dot{e}_i(t) \\ &\quad + a\frac{\delta_i^2}{2}q_i(t) + b_o\frac{\delta_i^2}{2}\tau_i(t) - \frac{\delta_i^2}{2}\ddot{q}_i^\star(t) \end{aligned} \quad (7)$$

The performance measure to find optimal control $\tau_i(t)$ which reduce the tracking error at the next time instant is chosen as

$$J_i = \frac{1}{2}\left[Q_i e_i^2(t+\delta_i) + R_i\tau_i^2\right] \qquad i=x,y \quad (8)$$

where $R_i \geq 0$ and $Q_i > 0$ are the weightings on the current control τ_i and the predicted output tracking error $e_i(t+\delta_i)$ respectively. Applying the necessary condition for minimization of J_i with respect to τ_i, i.e. $\frac{\partial J_i}{\partial \tau_i} = 0$ and assuming control does not saturate [22], control τ_i is obtained as

$$\tau_i = -\frac{1}{b_i}\left[e_i(t) + \delta_i\dot{e}_i(t) + a\frac{\delta_i^2}{2}q_i(t) - \frac{\delta_i^2}{2}\ddot{q}_i^\star(t)\right] \quad (9)$$

where

$$b_i = \frac{8R_i m^3 z_c^3 + 2Q_i m z_c \delta_i^4}{4Q_i m^2 z_c^2 \delta_i^2} \quad (10)$$

It can be verified that the sufficiency conditions of the optimization problem i.e. $\frac{\partial^2 J_i}{\partial \tau_i^2} > 0$ is satisfied [17].

Controller performance of (9) is driven by the design parameters, i.e., δ_i, Q_i and R_i hence they need to be selected properly. By replacing the generalised state variable q_i by (x,y) we get the predictive control for (x,y) axis.

III. UDE based Robust Predictive Control

A. Predictive Control in Presence of External Disturbance

The LIPM dynamics (3)-(4) with external disturbance d_i acting on it can be represented by

$$\begin{aligned} \ddot{q}_i &= aq_i + b_o\tau_i + d_i \quad (11) \\ Y_i &= q_i \qquad i=x,y \quad (12) \end{aligned}$$

The predicted output tracking error at time $(t+\delta_i)$ i.e. $e_i(t+\delta_i) = Y_i(t+\delta_i) - Y_i^\star(t+\delta_i)$ is

$$\begin{aligned} e_i(t+\delta_i) &= e_i(t) + \delta_i\dot{e}_i(t) + a\frac{\delta_i^2}{2}q_i(t) + b_o\frac{\delta_i^2}{2}\tau_i(t) \\ &\quad - \frac{\delta_i^2}{2}\ddot{q}_i^\star(t) + \frac{\delta_i^2}{2}d_i(t) \end{aligned} \quad (13)$$

Following the similar procedure presented in Section II-B, the output tracking predictive control in presence of external disturbance can be obtained as

$$\begin{aligned} \tau_i &= -\frac{1}{b_i}\left[e_i(t) + \delta_i\dot{e}_i(t)\right. \\ &\quad \left. + a\frac{\delta_i^2}{2}q_i(t) - \frac{\delta_i^2}{2}\ddot{q}_i^\star(t) + \frac{\delta_i^2}{2}d_i(t)\right] \end{aligned} \quad (14)$$

where constant b_i is as given in (10). To Implement controller (14), one requires model of the disturbance d_i. Since the disturbances are unknown, estimation of the same is needed. Here, disturbance is estimated using the Uncertainty and Disturbance Estimator (UDE) explained as follows.

B. UDE based Robust Continuous Time Predictive Control (UDE+CTPC)

Consider the LIPM dynamics given by (11). Substituting predictive control of (14) into (11) and solving for d_i one gets

$$\begin{aligned} d_i &= c_{di}\left[\ddot{q}_i(t) - \frac{g}{z_c}q_i(t) + \frac{1}{mz_c b_i}(e_i(t)\right. \\ &\quad \left. + \delta_i\dot{e}_i(t) + a\frac{\delta_i^2}{2}q_i(t) - \frac{\delta_i^2}{2}\ddot{q}_i^\star(t))\right] \end{aligned} \quad (15)$$

where,

$$c_{di} = \frac{2mz_c b_i}{2mz_c b_i - \delta_i^2}$$

In UDE based disturbance estimation technique, external disturbances acting on the system is estimated by using different filters [37]. Here, a first order filter [29] is used for estimating the disturbance as

$$G_{fi}(s) = \frac{1}{1+\tau_{f_i}s} \qquad i=x,y \quad (16)$$

where $\tau_{f_i} > 0$. The estimate of disturbance d_i, i.e., $\hat{d}_i$ is estimated by passing Eq. (15) through a low pass filter of (16) as

$$\hat{d}_i(s) = G_{f_i}(s)d_i(s) \quad (17)$$

Substituting $\hat{d}_i$ in place of d_i in (14), the UDE based robust predictive control (UDE+CTPC) takes the form of

$$\begin{aligned} \tau_i &= -\frac{1}{b_i}\left[e_i(t) + \delta_i\dot{e}_i(t)\right. \\ &\quad \left. + a\frac{\delta_i^2}{2}q_i(t) - \frac{\delta_i^2}{2}\ddot{q}_i^\star(t) + \frac{\delta_i^2}{2}\hat{d}_i(t)\right] \end{aligned} \quad (18)$$

Now considering the situation of no penalty on control, i.e., $R_i = 0,\ \ i=x,y$ and carrying out few mathematical simplifications, the controller (14) takes the form as

$$\tau_i = \frac{1}{b_o}\left[\ddot{q}_i^\star(t) - \frac{2}{\delta_i^2}e_i(t) - \frac{2}{\delta_i}\dot{e}_i(t) - aq_i(t) - d_i\right] \quad (19)$$

Now defining,$u_{a_i} = -aq_i$ controller of (19) can be written as

$$\tau_i = \frac{1}{b_o}\left[u_{a_i} + u_{d_i} + \nu_i\right] \tag{20}$$

where,

$$u_{d_i} = -\hat{d}_i \tag{21}$$

$$\nu_i = \ddot{q}_i^{\star}(t) - k_1 e_i(t) - k_2 \dot{e}_i(t) \tag{22}$$

where $k_1 = \frac{2}{\delta_i^2}$ and $k_2 = \frac{2}{\delta_i}$. Substituting (20) into (11) one gets

$$\ddot{q}_i = u_{d_i} + \nu_i + d_i \tag{23}$$

Hence d_i can be obtained as

$$d_i = \ddot{q}_i - u_{d_i} - \nu_i \tag{24}$$

Estimation of the external disturbance d_i with filter given by (16) is obtained as [29], [37]

$$d_i = \tau_{f_i}\dot{\hat{d}}_i + \hat{d}_i \tag{25}$$

Now substituting (21), (25) into (23) one gets

$$\tau_{f_i}\dot{\hat{d}}_i = \ddot{q}_i - \nu_i \tag{26}$$

Integrating both the sides of (26) gives

$$\hat{d}_i = \frac{1}{\tau_{f_i}}\dot{q}_i - \frac{1}{\tau_{f_i}}\int \nu_i dt \tag{27}$$

Now substituting (27) into (20) one gets

$$\tau_i = \frac{1}{b_o}\left[-aq_i + \nu_i - \frac{1}{\tau_{f_i}}\dot{q}_i - \frac{1}{\tau_{f_i}}\int \nu_i dt\right] \tag{28}$$

Note that with the proper choice of τ_{f_i}, $\hat{d}_i \approx d_i$. The control law of (28) is termed as the UDE based robust continuous time predictive control (UDE+CTPC). Controller (28) is implementable with the UDE estimated disturbance.

IV. Stability Analysis

As the dynamics of the $3-$D LIPM is decoupled, closed loop stability analysis as outlined in [21], [22] is presented in x-plane only. Here, the closed loop stability for the dynamics of (11) with $q_i = x$ and external disturbance d_x under the controller of (28) without penalty of control input, i.e,. $R_x = 0$ is presented. To this end, the system state vector is defined as $X_p = [x_1\ x_2]^T$, eq. (11) for the x-plane can be written as

$$\dot{X}_p = A_p X_p + B_p \tau_x + B_d d_x \tag{29}$$

where

$$A_p = \begin{bmatrix} 0 & 1 \\ a & 0 \end{bmatrix}; B_p = \begin{bmatrix} 0 \\ b_o \end{bmatrix}; B_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; A_o = \begin{bmatrix} 0 & 0 \\ -a & 0 \end{bmatrix}$$

Defining the reference state vector as $R_x = [x^{\star}\ \dot{x}^{\star}]^T$, and re-writing the UDE+CTPC controller (28) as

$$\tau_x = -K_p X_p + K_p R_x + K_R R_x - \frac{1}{b_o}\hat{d}_x + \frac{1}{b_o}\ddot{x}^{\star} \tag{30}$$

where the gain vectos are defined as $K_p = [\frac{a+k_1}{b_o} \quad \frac{k_2}{b_o}]$ and $K_R = [-\frac{a}{b_o}\ 0]$. The reference state vector dynamics, $\dot{R}_x$ is written as

$$\dot{R}_x = A_p R_x + A_o R_x + B_d \ddot{x}^{\star} \tag{31}$$

State vector tracking error is defined as $e_{cx} = R_x - X_p$. Using (29), (30) and (31) and carrying out few mathematical simplifications, the dynamics of the state tracking error is written as

$$\dot{e}_{cx} = (A_p - B_p K_p)e_{cx} - B_d \tilde{d}_x \tag{32}$$

disturbance estimation error is defined as $\tilde{d}_x = d_x - \hat{d}_x$. The dynamics of the disturbance estimation error is obtained by subtracting (25) from $\tau_{f_x}\dot{d}_x$ as

$$\tau_{f_x}\dot{d}_x - d_x = \tau_{f_x}\dot{d}_x - \tau_{f_x}\dot{\hat{d}}_x - \hat{d}_x \tag{33}$$

Rearranging (33) as

$$\dot{\tilde{d}}_x = -\frac{1}{\tau_{f_x}}\tilde{d}_x + \dot{d}_x \tag{34}$$

Combining (32) and (34), one can obtain the closed loop error dynamics as

$$\begin{bmatrix} \dot{e}_{cx} \\ \dot{\tilde{d}}_x \end{bmatrix} = \begin{bmatrix} (A_p - B_p K_p) & -B_d \\ 0 & -\frac{1}{\tau_{f_x}} \end{bmatrix}\begin{bmatrix} e_{cx} \\ \tilde{d}_x \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\dot{d}_x \tag{35}$$

From (35), it is seen that the system matrix is in a block traingular form and eigenvalues of (35) are given by solving

$$|sI - (A_p - B_p K_p)||s - (-\frac{1}{\tau_{f_x}})| = 0 \tag{36}$$

The pair $(A_p; B_p)$ is controllable, to obtain satisfactory tracking performance K_p need to be chosen appropriately. Next, to ensure stable error dynamics (35), $\tau_{f_x} > 0$ need to be chosen carefully. Lastly, if the rate of change of disturbance is small, i.e., if $\dot{d}_x \approx 0$, then the closed loop error dynamics (35) is asymptotically stable [32]. From (34), it is evident that the filter time constant, τ_{f_x}, is a critical factor in disturbance estimation.

V. Simulations and Results

Numerical simulations of the CTPC and UDE+CTPC are carried out and results are presented in this section. The simulation data for $3-$D LIPM representing *Mahru-I* humanoid is taken from [12] and is as given in Table I.

TABLE I

Simulation data

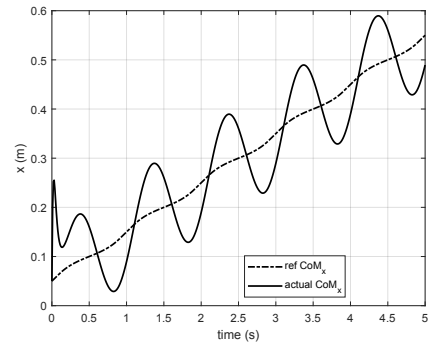
Parameter	Definition	Value
m	Robot mass	$67.68kg$
z_c	Constrained/CoM height of the robot	$0.678m$
g	Gravity acceleration	-$9.81m/s^2$
Q_x, Q_y	Error weighting matrix	1000000
R_x, R_y	Control weighting matrix	0
δ_x, δ_y	Prediction horizon	$0.1s$
τ_{fx}, τ_{fy}	time constant of UDE filter	$0.02s$
$[x(0), y(0)]$	Initial position	$[0.05;0.02]m$
$[\dot{x}(0), \dot{y}(0)]$	Initial velocity	$[0;0]m/s$

External disturbances of $d_x = 20sin(2\pi t)\ m/s^2$ and $d_y = 20cos(2\pi t)\ m/s^2$ are introduced in the LIPM model of (11). The reference CoM trajectories for the LIPM representing *Mahru-I* humanoid are designed for 5 sec of walking as given in [12]. The design parameters needed in predictive

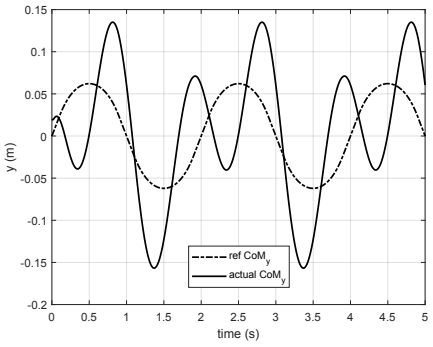
TABLE II

PERFORMANCE COMPARISON OF CTPC AND UDE+CTPC CONTROLLER

Controller	RMS error in x (m)	RMS error in y (m)
CTPC	0.0722	0.0690
UDE+CTPC	0.0061	0.0023



(a)



(b)

Fig. 2. CoM tracking performance of predictive control (CTPC) (a) x-plane (b) y-plane

controllers are as given in Table I. Performance of the CTPC controller of (9) is compared with the robust UDE+CTPC controller of (18) through simulations. The CoM tracking history of the CTPC control due to external disturbance is shown in Fig. 2(a)-(b) from where significant tracking error can be observed. However, as can be seen in Fig. 3(a)-(b), the performance of UDE+CTPC controller is satisfactory. Plots of the disturbance and it's UDE estimate presented in Fig. 3(c)-(d) shows that the disturbance d_i is estimated accurately by the UDE of (17). It is imperative to note that, in case of high frequency disturbances, the estimation accuracy with the use of filter (16) degrade and as discussed in [30] an appropriate filter need to be chosen to estimate the disturbances accurately. Root mean square error (eRMS) index [38] for the CTPC and UDE+CTPC controller is calculated using

$$e_{RMS} = \sqrt{\sum_{i=1}^{n} \frac{(x_i^\star - x_i)^2}{n}} \tag{37}$$

where, x_i is CoM and $x_i^\star$ is reference CoM both at ith sample, n is the number of samples. The eRMS index for both the controllers is presented in Table II. The eRMS for the UDE+CTPC based robust controller is significantly less than the nominal CTPC controller. To test the efficacy of the proposed controller in realistic scenarios, the porposed work will be extended through simulation studies using multi-physics model of the humanoid robot.

VI. CONCLUSION

A novel UDE based robust optimal predictive control (UDE+CTPC) for humanoid center of mass tracking is proposed. Closed-loop stability of the system is presented. From numerical simulations, it is demonstrated that the trajectory tracking performance of the novel UDE based robust predictive control is better than the nominal predictive control thus addressing the issue of external disturbances effectively.

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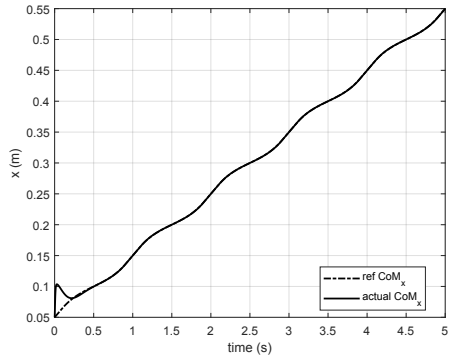
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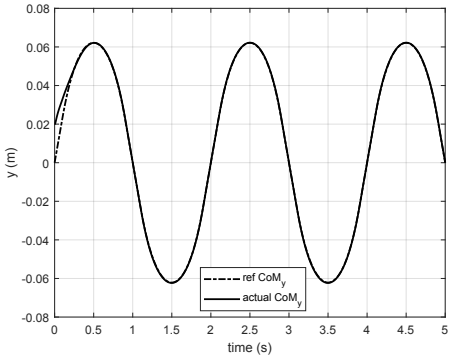
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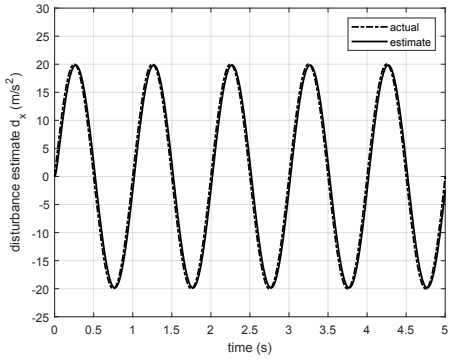
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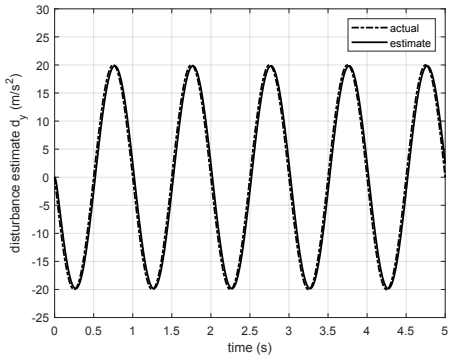
(a)



(b)



(c)



(d)

Fig. 3. Performance of UDE+CTPC control (a) and (b) CoM trajectory tracking (c) and (d) disturbance and it's estimate

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