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Adaptive Fuzzy Predictor based Fast Terminal Sliding Mode Controller Design for Two-link Robot Manipulator

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***Abstract*— This paper proposes an Adaptive Fuzzy Predictor based Fast Terminal Sliding Mode Control (AFP-FTSMC) for a two-link robotic manipulator (TLRM). The unknown disturbances, changing with time, present in the TLRM system are estimated using the fuzzy predictor model and the accurate control action is formulated using the FTSM controller to tackle the nonlinearities in the system. The disturbance, varying with time, is estimated using adaptive fuzzy predictor, and the fuzzy adaptive rule is developed from the Lyapunov stability criteria. The chattering phenomenon and singularity problem is efficiently addressed by utilizing the fast terminal sliding mode control law. The overall controller is capable of producing fast-tracking performance by rejecting the disturbance and chattering adequately. The stable performance of the TLRM system with the proposed controller is guaranteed by utilizing Lyapunov stability criteria. The experimental results demonstrate that the proposed controller achieves superior tracking performance and robustness for nonlinear TLRM system compared to the existing terminal sliding mode control method, even in the presence of significant uncertainties and time-varying external disturbances.**

I. Introduction

The field of robotics has experienced remarkable advancements, driven by the need for precise and efficient control systems that can handle the complex nonlinear dynamics, coupled with system uncertainties and external disturbances for robotic manipulator. In the recent years, various control methodologies have been explored to address these challenges. For instance, robust control techniques such as sliding mode control (SMC) have been widely investigated for their effectiveness in handling system uncertainties and external disturbances [1], [2]. The integration of fuzzy logic with traditional control strategies has shown significant improvements in adaptability and robustness, as fuzzy logic controllers can effectively manage nonlinearities and uncertainties in robotic systems [3], [4]. Disturbance observers have also been employed to estimate the disturbances which can be rejected by the controllers, as evidenced by recent works [5], [6]. Hybrid control strategies that combine elements of fuzzy logic, adaptive control, and disturbance observation have demonstrated enhanced performance in managing the dynamic behavior of robotic manipulators [7], [8]. Intelligent control algorithms, which merge machine learning techniques with traditional approaches, are continually evolving to enhance adaptability and performance in uncertain environments [9], [10].

The conventional SMC maintains system stability and robustness by guiding the system state to a predetermined sliding surface, it faces challenges in ensuring robustness during the reaching phase [11]. Additionally, it is susceptible to the chattering phenomenon, which can deteriorate system performance [12]. To address the limitations of conventional SMC, terminal sliding mode control (TSMC) and its variants, such as fast terminal sliding mode control (FTSMC), have been introduced. TSMC enhances precision by ensuring finite-time convergence to the sliding surface, and addresses chattering [13], [14], while FTSMC further accelerates the convergence rate without encountering singularity issues [15], [16]. This approach not only improves tracking performance but also mitigates the chattering effect associated with conventional SMC [17]. Though the performance of FTSMC degrades in presence of time-varying external disturbances present in the practical system.

Incorporating fuzzy logic systems into sliding mode control frameworks has demonstrated significant improvements in handling uncertainties and reducing chattering. Fuzzy logic systems provide a mechanism to approximate unknown nonlinearities and uncertainties within the system, thereby enhancing control accuracy [18]. Adaptive fuzzy logic systems (AFLS) dynamically adjust the control parameters in real-time, further improving system robustness and performance [19], [20]. Though to accurately reject the

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disturbance, proper estimate of uncertainties and disturbance, varying with time are essential. It is also necessary to track the desired trajectory fast and properly for nonlinear system.

Motivated from the drawbacks of the existing methodologies, this paper proposes an adaptive fuzzy predictor based fast terminal sliding mode controller (AFP-FTSMC) for a TLRM. The time-varying disturbances in the TLRM is estimated using the fuzzy predictor model and the accurate control action is formulated using the FTSM controller to handeled the nonlinearities present in TLRM. To obtain proper estimation of the disturbance, the predictor model is formulated using the fuzzy logic. The disturbance is predicted adaptively to cope with any disturbances, varies with time suddenly appears in the system. The fuzzy adaptive law to formulate the estimation of disturbance is developed from the Lyapunov stability criteria. Therefore, it guarantees stable adaptive estimates of the disturbances. Then, the FTSM controller is developed to cancel out them properly and to achieve efficient tracking performance. The singularity problem which may cause instability of the practical system has also been addressed utilizing the FTSM controller. For practical system, it is essential to have fast estimation and rejection of disturbances which is effectively achieved utilizing the proposed controller. Therefore, the proposed controller is capable of producing fast tracking performance by nullifying the time-varying disturbances present in the system. The overall stability of the proposed AFP-FTSMC is guaranteed utilizing Lyapunov stability theory. The effectiveness of the proposed AFP-FTSMC is validated on a two-link robotic manipulator experimental setup. The experimental results demonstrate that the proposed controller achieves superior tracking performance and robustness compared to existing control methods, even in the presence of significant uncertainties and external disturbances.

Therefore, the contributions of the proposed methodology can be summarized as follows.

- The adaptive fuzzy predictor of the proposed AFP-FTSMC can estimate the disturbances present in the experimental TLRM system accurately. The boundedness of the adaptive estimates of the disturbance is also guaranteed utilizing Lyapunov stability criteria.
- The fast terminal sliding mode controller of the proposed AFP-FTSMC is producing the efficient tracking performance by reducing the chattering problem and nullifying the disturbances. The overall AFP-FTSMC is proved to be stable with the help of Lyapunov stability criteria.
- The effectiveness of the proposed control methodology is validated on a TLRM experimental setup.

The rest of the paper is arranged as follows. At first, the problem formulation is provided in section II. The main results are described in section III with subsection describing designing of adaptive fuzzy predictor and designing, fast terminal sliding mode controller and the stability analysis of the overall proposed methodology. Section IV portrays the experimental results and then the concluding remarks with future scope are provided in Section V.

II. Problem Formulation

A. Description of the system model

A planar two-link robot manipulator (TLRM) which moves in the gravitational plane with two degree of freedom (DoF), as shown in Fig. 1, is described in this section. The

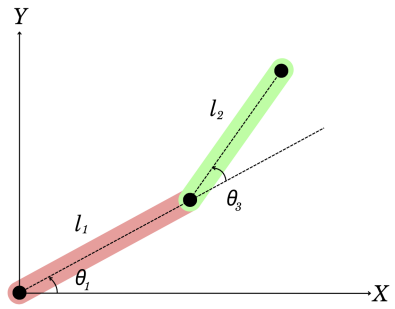


Fig. 1. Kinematic diagram of TLRM

system dynamics can be formulated from the dynamics and reverse kinematics model as follows:

$$\ddot{\theta}_1 = \frac{1}{((m_1+m_2)l_1^2+m_2l_2^2)}\big[\tau_1-(m_2l_2^2+m_2l_1l_2cos\theta_3)\dot{\theta}_4 + m_2l_1l_2sin\theta_3\theta_4^2+2m_2l_1l_2\theta_2\theta_4sin\theta_3 - (m_1+m_2)gl_1cos\theta_1 - m_2gl_2cos(\theta_1+\theta_3)+\mathcal{C}_1\mathfrak{d}_1\big] \quad (1)$$

$$\ddot{\theta}_3 = \frac{1}{m_2l_2^2}\big[\tau_2-(m_2l_2^2+m_2l_1l_2cos\theta_3)\dot{\theta}_2 - m_2l_1l_2sin\theta_3\theta_2^2+m_2gl_2cos(\theta_1+\theta_3)+\mathcal{C}_2\mathfrak{d}_2\big] \quad (2)$$

where m_i, l_i with $i=[1,2]$ are the mass and length of the links. θ_1 is the angle between base and first link, i.e., angular position of the first link, $\theta_2=\dot{\theta}_1$ is the angular velocity of the first link, θ_3 is the angle between first and second link, i.e., angular position of the second link, $\theta_4=\dot{\theta}_3$ is the angular velocity of the second link, $\underline{\theta}=[\theta_1,\theta_2,\theta_3,\theta_4]^T$ are the states of the system. $\mathcal{C}_1$ and $\mathcal{C}_2$ are constant scaling factors of the fuzzy rules, $\mathfrak{d}_1$ and $\mathfrak{d}_2$ are the disturbances present in the first and second link respectively. τ_i is the imparted joint torque with counterclockwise rotation assumed to be positive, i.e., the inputs of the system.

Again this system model can be represented as:

$$\dot{\underline{\theta}} = A\underline{\theta}+B(\tau+\underline{\mathcal{C}}\underline{\mathfrak{d}}) \quad (3)$$

where, $A\in R^{4x4}$ is the system matrix, $B\in R^{4x2}$ is the input matrix, $\underline{\mathcal{C}}=[0,\mathcal{C}_1,0,0;0,0,0,\mathcal{C}_2]\in R^{2x4}$ is the constant matrix, $\underline{\mathfrak{d}}=[0,\mathfrak{d}_1,0,\mathfrak{d}_2]^T\in R^{4x1}$ is the disturbance matrix.

The objective is to design a fuzzy predictor which can be efficiently estimate the time-varying disturbances present in the system. Therefore, in this research work, a fuzzy disturbance observer has been designed. After proper estimation of the disturbance, it has to be nullified properly utilizing efficient controller. With this aim, in this research work, a fast terminal sliding mode controller is designed. These design procedures are elaborated in the next section.

III. Main Results

To eliminate the disturbances, varies with time, present in the nonlinear TLRM system and to design efficient tracking controller, in this proposed work, an adaptive fuzzy predictor based fast terminal sliding mode control (AFP-FTSMC) methodology is designed. The architecture of the proposed controller is provided in Fig. 2. At first, the adaptive fuzzy logic based predictor is designed to properly estimate the time-varying disturbances present in the system. Then, to eliminate those and to generate accurate tracking performance, fast terminal sliding mode controller is designed. The overall proposed controller is capable of tackling system with time-varying disturbances, uncertainties, nonlinearities efficiently.

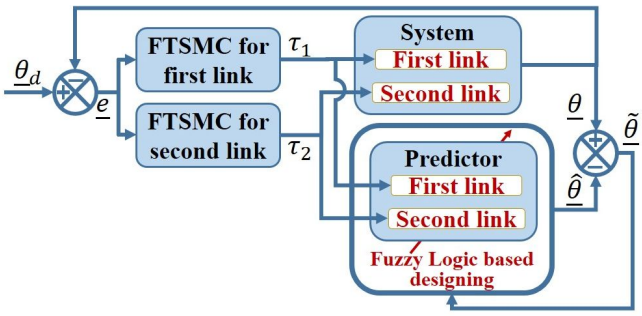


Fig. 2. Architecture of proposed AFP-FTSMC

A. Designing of adaptive fuzzy predictor

In this research work, one of the main objective is to estimate the time-varying disturbances present in the system. Therefore, a predictor model is designed utilizing universal approximator T-S fuzzy system. The unknown time-varying disturbances related to the first and second link i.e. $\mathfrak{d}_1$ and $\mathfrak{d}_2$ respectively can be estimated utilizing fuzzy logic system. $\mathcal{Q}$ numbers of fuzzy *IF-THEN* rules are augmented together to formulate the overall fuzzy predictor. Now, the predictor model dynamics can be formulated with the help of the *IF-THEN* rules.

For, Rule q: *IF* $\tilde{\theta}_1(t)$ is E_1^q, $\tilde{\theta}_2(t)$ is E_2^q, ..., $\tilde{\theta}_n(t)$ is E_n^q, *THEN*: The dynamics of the predictor as the consequence part of the fuzzy logic system can be given as:

$\ddot{\hat{\theta}}_1 = \frac{1}{((m_1+m_2)l_1^2+m_2l_2^2)}\big[\tau_1 - (m_2l_2^2 + m_2l_1l_2cos\hat{\theta}_3)\dot{\hat{\theta}}_4$
$+ m_2l_1l_2sin\hat{\theta}_3\hat{\theta}_4^2 + 2m_2l_1l_2\hat{\theta}_2\hat{\theta}_4sin\hat{\theta}_3$
$- (m_1+m_2)gl_1cos\hat{\theta}_1 - m_2gl_2cos(\hat{\theta}_1+\hat{\theta}_3) + \mathcal{C}_1\hat{\mathfrak{d}}_1\big]$ and
$\ddot{\hat{\theta}}_3 = \frac{1}{m_2l_2^2}\big[\tau_2 - (m_2l_2^2 + m_2l_1l_2cos\hat{\theta}_3)\dot{\hat{\theta}}_2$
$- m_2l_1l_2sin\hat{\theta}_3\hat{\theta}_2^2 + m_2gl_2cos(\hat{\theta}_1+\hat{\theta}_3) + \mathcal{C}_2\hat{\mathfrak{d}}_2\big]$ where $\underline{\hat{\theta}} = [\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4]^T$ are the states of the predictor, $\underline{\tilde{\theta}}(t) = [\underline{\theta} - \underline{\hat{\theta}}] = [\tilde{\theta}_1(t), \tilde{\theta}_2(t), ..., \tilde{\theta}_n(t)]^T$ are represents premise variables of the fuzzy rules, $\underline{E}_r^q = [E_1^q, E_2^q, ..., E_n^q]$ is the fuzzy set with $r = 1, 2, ..., n$.

After applying de-fuzzification, the overall fuzzy predictor model of the TLRM can be given as:

$$\ddot{\hat{\theta}}_1 = \frac{1}{((m_1+m_2)l_1^2+m_2l_2^2)}\big[\tau_1 - (m_2l_2^2 + m_2l_1l_2cos\hat{\theta}_3)\dot{\hat{\theta}}_4 + m_2l_1l_2sin\hat{\theta}_3\hat{\theta}_4^2 + 2m_2l_1l_2\hat{\theta}_2\hat{\theta}_4sin\hat{\theta}_3 - (m_1+m_2)gl_1cos\hat{\theta}_1 - m_2gl_2cos(\hat{\theta}_1+\hat{\theta}_3) + \sum_{q=1}^{\mathcal{Q}}\underline{\xi}_q\mathcal{C}_1\hat{\mathfrak{d}}_{1q}\big] \tag{4}$$

$$\ddot{\hat{\theta}}_3 = \frac{1}{m_2l_2^2}\big[\tau_2 - (m_2l_2^2 + m_2l_1l_2cos\hat{\theta}_3)\dot{\hat{\theta}}_2 - m_2l_1l_2sin\hat{\theta}_3\hat{\theta}_2^2 + m_2gl_2cos(\hat{\theta}_1+\hat{\theta}_3) + \sum_{q=1}^{\mathcal{Q}}\underline{\xi}_q\mathcal{C}_2\hat{\mathfrak{d}}_{2q}\big] \tag{5}$$

where the fuzzy basis function is defined as:

$\xi_q = \frac{\prod_{r=1}^{n}\mu_{E_r^q}(\hat{e}_r)}{\sum_{q=1}^{N}\prod_{r=1}^{n}\mu_{E_r^q}(\hat{e}_r)}$

$\mu_{E_r^q}(\hat{e}_r)$ is the membership of the function $\hat{e}_r$ in $\mu_{E_r^q}$. Similar to (3), the predictor model with fuzzy disturbance estimation can be written as: $\dot{\underline{\hat{\theta}}} = A\underline{\hat{\theta}} + B\big(\tau + \sum_{q=1}^{\mathcal{Q}}\underline{\xi}_q\underline{\mathcal{C}\hat{\mathfrak{d}}}_q\big)$. where, $\underline{\hat{\mathfrak{d}}} = [0, \hat{\mathfrak{d}}_1, 0, \hat{\mathfrak{d}}_2]^T \in R^{4x1}$ is the matrix with estimated disturbances.

The difference between the system states and the states of the fuzzy predictor can be given as $\dot{\underline{\tilde{\theta}}} = \dot{\underline{\hat{\theta}}} - \dot{\underline{\theta}}$. Therefore, the error dynamics can be written as:

$$\dot{\underline{\tilde{\theta}}} = A\underline{\tilde{\theta}} + B\big(\sum_{q=1}^{\mathcal{Q}}\underline{\xi}_q\underline{\mathcal{C}\hat{\mathfrak{d}}}_q - \underline{\mathcal{C}\mathfrak{d}}\big) \tag{6}$$

Theorem 1. *If the error dynamics between the system model and fuzzy predictor model can be given as* (6) *then, the fuzzy predictor model will be stabilized if the fuzzy adaptive estimate of the disturbance can be formulated using the following rule:*

$$\sum_{q=1}^{\mathcal{Q}}\dot{\underline{\hat{\mathfrak{d}}}}_q = -\Gamma\underline{\tilde{\theta}}^T PB\underline{\mathcal{C}}^{-1}\underline{\xi}_q^{-1} \tag{7}$$

Proof: Let, the Lyapunov candidate is:

$$V_o = \frac{1}{2}\underline{\tilde{\theta}}^T P\underline{\tilde{\theta}} + \big(\sum_{q=1}^{\mathcal{Q}}\underline{\xi}_q\underline{\mathcal{C}\hat{\mathfrak{d}}}_q - \underline{\mathcal{C}\mathfrak{d}}\big)^T\Gamma^{-1}\big(\sum_{q=1}^{\mathcal{Q}}\underline{\xi}_q\underline{\mathcal{C}\hat{\mathfrak{d}}}_q - \underline{\mathcal{C}\mathfrak{d}}\big) \tag{8}$$

where P is the solution of algebraic Lyapunov equation $A^TP + PA = -Q$ with P and Q being positive definite matrix.

Now, taking derivative of (8), considering, $\sum_{q=1}^{\mathcal{Q}}\underline{\xi}_q\underline{\hat{\mathfrak{d}}}_q - \underline{\mathfrak{d}} = \underline{\tilde{\mathfrak{d}}}$ and putting the value of $\dot{\underline{\tilde{\theta}}}$ from (6) it can be found that:

$\dot{V}_o = \frac{1}{2}(A\underline{\tilde{\theta}})^T P\underline{\tilde{\theta}} + \frac{1}{2}\underline{\tilde{\theta}}^T PA\underline{\tilde{\theta}} + \underline{\tilde{\theta}}^T PBC\big(\sum_{q=1}^{\mathcal{Q}}\underline{\xi}_q\underline{\mathcal{C}\hat{\mathfrak{d}}}_q - \underline{\mathcal{C}\mathfrak{d}}\big)$
$+ \underline{\mathcal{C}\tilde{\mathfrak{d}}}^T\Gamma^{-1}\big(\sum_{q=1}^{\mathcal{Q}}\underline{\xi}_q\underline{\mathcal{C}\dot{\hat{\mathfrak{d}}}}_q - \underline{\mathcal{C}\dot{\mathfrak{d}}}\big)$

Now to stabilize the predictor following condition should holds: $\dot{V}_o = -\underline{\tilde{\theta}}^T Q\underline{\tilde{\theta}} - \underline{\mathcal{C}\tilde{\mathfrak{d}}}^T\Gamma^{-1}\underline{\mathcal{C}\dot{\mathfrak{d}}}$. It incurs:

$$\underline{\tilde{\theta}}^T PB\underline{\mathcal{C}\tilde{\mathfrak{d}}} + \underline{\mathcal{C}\tilde{\mathfrak{d}}}^T\Gamma^{-1}\sum_{q=1}^{\mathcal{Q}}\underline{\xi}_q\underline{\mathcal{C}\dot{\hat{\mathfrak{d}}}}_q = 0 \tag{9}$$

Solution of (9) provides (7) which proves the theorem.

From this theorem it can be stated that, $\dot{V}_o$ is negative definite, i.e., $\dot{V}_o < 0$. Therefore, the predictor will be stable provided the disturbances are bounded.

After estimating the disturbance by using the fuzzy predictor, fast terminal sliding mode controller is designed to obtain satisfactory control performance which is elaborated in the next section.

B. Designing of fast terminal sliding mode controller

To control practical system, it is essential to have fast tracking performance. Therefore, in this research work, fast terminal sliding mode controller is designed for a nonlinear MIMO TLRM system.

The sliding surface for the first link is considered as:

$$\mathfrak{S}_{12} = \epsilon_1 e_1 + \epsilon_2 e_2 + \kappa_1 e_1^{p/q} + \kappa_2 e_2^{p/q} \tag{10}$$

where $e_1 = \theta_{d1} - \theta_1, e_2 = \theta_{d2} - \theta_2$ are the errors of the first and second state respectively, $\theta_{d1} \in R$ and $\theta_{d2} = 0$ are the reference input to the first and second state respectively. $\epsilon_1 > 0, \epsilon_2 > 0, \kappa_1 > 0, \kappa_2 > 0$ are the design constants. To design the fast terminal sliding mode controller, p and q are chosen as odd integer numbers satisfying the condition $1 < \frac{p}{q} < 2$. If the $x(0)$ is relatively away from equilibrium, the linear part, $\epsilon_1 e_1$ and $\epsilon_2 e_2$, will drive perturbed states to converge equilibrium in finite-time with suitably chosen initial states whereas the nonlinear terms, $e_1^{p/q}$ and $e_2^{p/q}$ help to ramp-up the error convergence rate to equilibrium.
Similarly, the sliding surface for the second link is:

$$\mathfrak{S}_{34} = \epsilon_3 e_3 + \epsilon_4 e_4 + \kappa_3 e_3^{p/q} + \kappa_4 e_4^{p/q} \tag{11}$$

where $e_3 = \theta_{d3} - \theta_3, e_4 = \theta_{d4} - \theta_4$ are the errors of the third and fourth state respectively, $\theta_{d3} \in R$ and $\theta_{d4} = 0$ are the reference input to the third and fourth state respectively. Therefore, $\underline{\theta}_d = [\theta_{d1}, \theta_{d2}, \theta_{d3}, \theta_{d4}]$ is the overall reference trajectory. $\epsilon_3 > 0, \epsilon_4 > 0, \kappa_3 > 0, \kappa_4 > 0$ are the design constants.

Theorem 2. *If the sliding surface for the first link can be given by* (10) *then, the system will be stable with the fast terminal sliding mode control law which can be given as:*

$$\begin{aligned}\tau_1 = \frac{((m_1+m_2)l_1^2 + m_2 l_2^2)}{\kappa_2 \frac{p}{q} e_2^{p/q-1}} \big[& \epsilon_1 \dot{e}_1 - \epsilon_2 \dot{\theta}_2 \\ & + \kappa_1 \frac{p}{q} e_1^{p/q-1} \dot{e}_1 + \mathfrak{K}_1 \mathfrak{S}_{12} + \mathfrak{K}_2 sgn(\mathfrak{S}_{12}) \big] \\ & - (m_2 l_2^2 + m_2 l_1 l_2 cos\theta_3)\dot{\theta}_4 \\ & + m_2 l_1 l_2 sin\theta_3 \theta_4^2 + 2 m_2 l_1 l_2 \theta_2 \theta_4 sin\theta_3 \\ & - (m_1+m_2) g l_1 cos\theta_1 - m_2 g l_2 cos(\theta_1+\theta_3) + \mathcal{C}_1 \hat{\mathfrak{d}}_1 \end{aligned} \tag{12}$$

where $\mathfrak{K}_1$, $\mathfrak{K}_2$ and $\mathcal{C}_1$ are the design constants.

Proof: Taking derivative of (10) and by putting the value of $\dot{\theta}_2$ from (1) it can be obtained that:

$$\begin{aligned}\dot{\mathfrak{S}}_{12} = {} & \epsilon_1 \dot{e}_1 - \epsilon_2 \dot{\theta}_2 + \kappa_1 \frac{p}{q} e_1^{p/q-1} \dot{e}_1 \\ & - \frac{\kappa_2 \frac{p}{q} e_2^{p/q-1}}{((m_1+m_2)l_1^2 + m_2 l_2^2)} \big[\tau_1 - (m_2 l_2^2 + m_2 l_1 l_2 cos\theta_3)\dot{\theta}_4 \\ & + m_2 l_1 l_2 sin\theta_3 \theta_4^2 + 2 m_2 l_1 l_2 \theta_2 \theta_4 sin\theta_3 \\ & - (m_1+m_2) g l_1 cos\theta_1 - m_2 g l_2 cos(\theta_1+\theta_3) + \mathcal{C}_1 \mathfrak{d}_1 \big] \end{aligned} \tag{13}$$

where $\dot{e}_1 = \dot{\theta}_{d1} - \dot{\theta}_1, \dot{e}_2 = -\dot{\theta}_2$.
Now by solving this for τ_1, (12) can be found out.

Theorem 3. *If the sliding surface for the second link can be given by* (11) *then, the system will be stable with the fast terminal sliding mode control law which can be given as:*

$$\begin{aligned}\tau_2 = \frac{m_2 l_2^2}{\kappa_4 \frac{p}{q} e_4^{p/q-1}} \big[& \epsilon_3 \dot{e}_3 - \epsilon_4 \dot{\theta}_4 + \kappa_3 \frac{p}{q} e_3^{p/q-1} \dot{e}_3 \\ & + \mathfrak{K}_3 \mathfrak{S}_{34} + \mathfrak{K}_4 sgn(\mathfrak{S}_{34}) \big] \\ & - (m_2 l_2^2 + m_2 l_1 l_2 cos\theta_3)\dot{\theta}_2 \\ & - m_2 l_1 l_2 sin\theta_3 \theta_2^2 + m_2 g l_2 cos(\theta_1+\theta_3) + \mathcal{C}_2 \hat{\mathfrak{d}}_2 \end{aligned} \tag{14}$$

where $\mathfrak{K}_2$, $\mathfrak{K}_3$ and $\mathcal{C}_2$ are the design constants.

Proof: Taking derivative of (11) and by putting the value of $\dot{\theta}_4$ from (2) it can be obtained that:

$$\begin{aligned}\dot{\mathfrak{S}}_{34} = {} & \epsilon_3 \dot{e}_3 - \epsilon_4 \dot{\theta}_4 + \kappa_3 \frac{p}{q} e_3^{p/q-1} \dot{e}_3 \\ & - \frac{\kappa_4 \frac{p}{q} e_4^{p/q-1}}{m_2 l_2^2} \big[\tau_2 - (m_2 l_2^2 + m_2 l_1 l_2 cos\theta_3)\dot{\theta}_2 \\ & - m_2 l_1 l_2 sin\theta_3 \theta_2^2 + m_2 g l_2 cos(\theta_1+\theta_3) + \mathcal{C}_2 \mathfrak{d}_2 \big] \end{aligned} \tag{15}$$

where $\dot{e}_3 = \dot{\theta}_{d3} - \dot{\theta}_3, \dot{e}_4 = -\dot{\theta}_4$.
Now by solving this for τ_2, (14) can be found out.

Now, the stability of the proposed controller has been analysed in the next sub-section.

C. Stability analysis

Let, the Lyapunov candidate vector for the first sliding surface is considered as:

$$V(\mathfrak{S}_{12}) = \frac{1}{2} \mathfrak{S}_{12}^T \gamma^{-1} \mathfrak{S}_{12} \tag{16}$$

For this sliding surface, the stability analysis of the first link is provided in the following theorem.

Theorem 4. *The response of the first link will be stabilized on the sliding surface* (10) *if the following condition takes place:*

$$\begin{aligned}\dot{V}(\mathfrak{S}_{12}) = \mathfrak{S}_{12}^T \gamma^{-1} \Big[& - \mathfrak{K}_1 \mathfrak{S}_{12} - \mathfrak{K}_2 sgn(\mathfrak{S}_{12}) \\ & - \mathcal{C}_1 \parallel \hat{\mathfrak{d}}_1 - \mathfrak{d}_1 \parallel \Big] \end{aligned} \tag{17}$$

Proof: Taking derivative of (16), and putting the values of $\dot{\mathfrak{S}}_{12}$ and then τ_1, one can find out:

$$\dot{V}(\mathfrak{S}_{12}) = \mathfrak{S}_{12}^T\gamma^{-1}\Big[\epsilon_1\dot{e}_1 - \epsilon_2\dot{\theta}_2 + \kappa_1\tfrac{p}{q}e_1^{p/q-1}\dot{e}_1 - \frac{\kappa_2\frac{p}{q}e_2^{p/q-1}}{((m_1+m_2)l_1^2+m_2l_2^2)}\Big[\frac{((m_1+m_2)l_1^2+m_2l_2^2)}{\kappa_2\frac{p}{q}e_2^{p/q-1}}\big[\epsilon_1\dot{e}_1 - \epsilon_2\dot{\theta}_2 + \kappa_1\tfrac{p}{q}e_1^{p/q-1}\dot{e}_1 + \mathfrak{K}_1\mathfrak{S}_{12} + \mathfrak{K}_2 sign(\mathfrak{S}_{12})\big] - (m_2l_2^2 + m_2l_1l_2cos\theta_3)\dot{\theta}_4 + m_2l_1l_2sin\theta_3\theta_4^2 + 2m_2l_1l_2\theta_2\theta_4 sin\theta_3 - (m_1+m_2)gl_1cos\theta_1 - m_2gl_2cos(\theta_1+\theta_3) + \mathcal{C}_1\hat{\mathfrak{d}}_1 - (m_2l_2^2 + m_2l_1l_2cos\theta_3)\dot{\theta}_4 + m_2l_1l_2sin\theta_3\theta_4^2 + 2m_2l_1l_2\theta_2\theta_4 sin\theta_3 - (m_1+m_2)gl_1cos\theta_1 - m_2gl_2cos(\theta_1+\theta_3) + \mathcal{C}_1\mathfrak{d}_1\Big]\Big]$$

Simplification of the above equation provides (17).

From this theorem it can be stated that, the $\hat{\mathfrak{d}}_1$ will reach $\mathfrak{d}_1$ within a finite time. This incurs that, the proposed method can properly estimate the disturbances present in the system and can provide efficient tracking control by eradicating that.

Similarly, the Lyapunov candidate for the second sliding surface can be considered as:

$$V(\mathfrak{S}_{34}) = \frac{1}{2}\mathfrak{S}_{34}^T\gamma^{-1}\mathfrak{S}_{34} \tag{18}$$

The stability of the system for this sliding surface has been analysed in the following theorem.

Theorem 5. *The response of the second link will be stabilized on the sliding surface* (11) *if the following condition holds:*

$$\dot{V}(\mathfrak{S}_{34}) = \mathfrak{S}_{34}^T\gamma^{-1}\Big[-\mathfrak{K}_1\mathfrak{S}_{34} - \mathfrak{K}_2 sgn(\mathfrak{S}_{34}) - \mathcal{C}_2 \parallel \hat{\mathfrak{d}}_2 - \mathfrak{d}_2 \parallel\Big] \tag{19}$$

Proof: Derivative of (18) incurs:

$$\dot{V}(\mathfrak{S}_{34}) = \mathfrak{S}_{34}^T\gamma^{-1}\dot{\mathfrak{S}}_{34} \tag{20}$$

Now, putting the value of $\dot{\mathfrak{S}}_{34}$, then τ_2, and after simplification, one can obtain (19).

IV. Experimental Results

A. Description of experimental Setup

The two-link robot manipulator (TLRM) used in this study was designed in CAD, 3D printed and assembled, as shown in Fig. 3. Each link is actuated using a servo motor which has a standard three pin plug. Two pins are for power and ground, and the third connector carries the control signal. The control signals are generated in the PC using MATLAB®and provided to the TLRM using a microcontroller, i.e., Arduino®Uno©. It sends pulse width modulated (PWM) signals to the control pin of the servo motor and the duration of pulse, i.e. the pulse width determines the angle of actuation while the frequency determines the speed. An inbuilt potentiometer is connected to the output shaft of the servo motor to obtain the feedback corresponds to the angular position.

At first the nominal parameters of the system has found out by using system identification. The lengths of the first and second links are $L_1 = 100$mm and $L_2 = 80$mm respectively. The masses of the first and second links along with actuating motors are $m_1 = 117$g and $m_2 = 58$g respectively. This system is controlled using the proposed methodology.

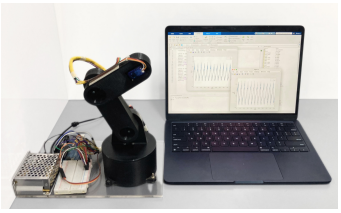


Fig. 3. Experimental setup of the TLRM system

B. Results

The proposed AFP-FTSMC is designed for the TLRM system consisting of time-varying disturbance and significant nonlinearities in it. The design parameters of the controllers and scaling factors are tuned by using particle swarm optimization (PSO) technique. Then tuned controllers are utilized to track the desired trajectories $\theta_{d1} = 1.6 + 0.5sin(0.15t)$ and $\theta_{d3} = 1.4 + 0.7sin(0.15t)$ by nullifying the time-varying uncertainties and disturbances. The response of the experimental setup with proposed control law is compared with terminal sliding mode control (TSMC) law. The comparison of the proposed AFP-FTSMC with TSMC in terms of integral absolute errors ($IAE = \int_0^t |\underline{e}(t)|dt$) are provided in Table I. The comparative tracking response of the first link of TLRM is provided in Fig. 4 and that for the second link is shown in Fig. 5. From the results, it can be seen that, the TSMC has significant chattering in the response, poor disturbance rejection ability as well as its tracking phenomenon is also not satisfactory. In the contrary, the proposed AFP-FTSMC is capable of producing efficient tracking performance for the nonlinear TLRM system by rejecting the disturbance and chattering. The comparative results of the control signal for the first and second link are depicted in Fig. 6 and Fig. 7 respectively. From results, it can be concluded that the chattering problem can be significantly eliminated in case of proposed methodology. The control signal developed is smooth enough than the TSMC in case of the proposed AFP-FTSMC.

TABLE I

Comparison of different control methodologies

	Integral absolute error (rad)	
Control Methodologies	**1st Link**	**2nd Link**
TSMC	81.3275	101.4513
Proposed AFP-FTSMC	48.3007	51.6172

V. Conclusion

In this research work, an adaptive fuzzy predictor based fast terminal sliding mode controller (AFP-FTSMC) is proposed for the TLRM experimental system consisting of nonlinearities and time-varying disturbances. The results show that, the adaptive fuzzy predictor can adequately estimate the time-varying disturbance present in the TLRM system. At the same time, the fast terminal sliding mode controller is capable of reducing the chattering phenomenon and producing efficient tracking performance by eliminating the estimated

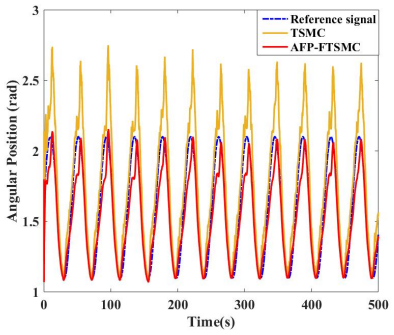


Fig. 4. Comparative experimental tracking response of the first link

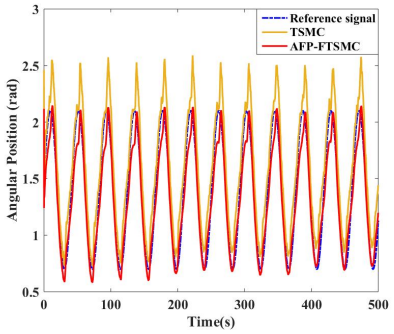


Fig. 5. Comparative experimental tracking response of the second link

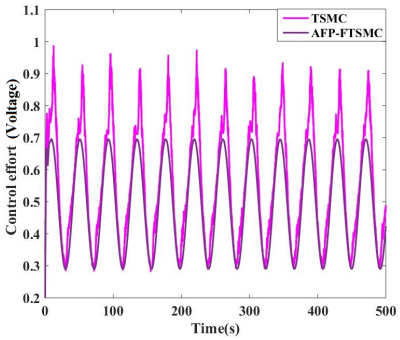


Fig. 6. Comparative control signal of the first link

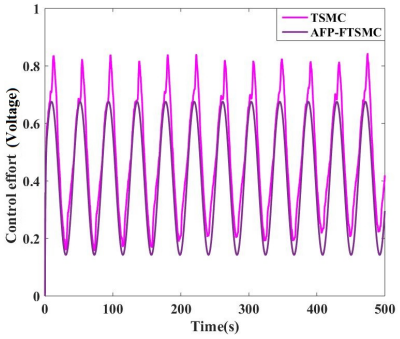


Fig. 7. Comparative control signal of the second link

disturbances for the nonlinear system. To demonstrate the superiority of the proposed methodology, it is compared with the terminal sliding mode controller which is incapable of rejecting the disturbances of the system, and its tracking performance is not satisfactory following undeniable time-varying disturbances.

In future, selectively adaptive controllers can be developed to achieve energy-efficient performance of testbeds.

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