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A new observer-based non-singular integral terminal sliding mode control for a second-order chaotic system

Rahash Nathasarma[1], Ankit Tiwari[1], Shilalipi Sahoo[1], Binoy Krishna Roy[1]

***Abstract*—This paper proposes a new non-singular integral terminal sliding mode control (NIT-SMC) for a class of second-order chaotic systems. To resolve the singularity issues in the integral terminal sliding mode control (IT-SMC), the sliding surface is redesigned and a nonsingular control law is proposed. A super-twisting sliding mode observer (STO) is used to observe the unmeasured state and disturbance of the system. The stability of the proposed controller is analysed using the Lyapunov stability concept. The performance of the proposed super-twisting observer-based non-singular integral terminal sliding mode controller (STO-NIT-SMC) is tested with the second-order Duffing chaotic system. Tracking of the desired square and sinusoidal trajectories using the proposed NIT-SMC is successfully achieved, and its superiority is established by comparing the performance with the IT-SMC and the existing NIT-SMC.**

***Keywords*—Integral terminal sliding mode, super-twisting sliding mode observer, chaotic system.**

I. Introduction

Most real-life systems are nonlinear. Designing control for nonlinear systems is challenging and complex due to the inherent complexities and unpredictable behaviors of nonlinearity. In the literature, sliding mode control (SMC) has been a popular choice due to its robustness to unknown disturbances and parameter uncertainties[1]. Different variants of SMCs have been proposed in recent years for a wide range of applications.

Integral sliding mode control (I-SMC) is preferred for two main reasons. First, the states are robust to disturbances and follow the sliding surface from the initial states. So, there is no reaching phase, only the sliding phase. Next, unlike other SMC variants, the states follow the same trajectory to reach the origin of different disturbances.

Due to this robustness, I-SMC is used for practical systems. In [2], the I-SMC achieved the desired states under the influences of both matched and unmatched disturbances. I-SMC is used in many practical applications like the active suspension system [3], the magnetic levitation system [4], etc.

In the literature, integral terminal sliding mode control (IT-SMC) has been reported recently to attain the desired behavior within a finite time. Along with robustness, IT-SMC helps to achieve a finite-time convergence. In [5], for quadrotors; in [6], for upper-limb rehabilitation exoskeleton; and in [7], for automobile electronic throttle, an IT-SMC is designed.

[1]National Institute of Technology Silchar, Silchar, 788010, Assam, India rahash21_rs@ee.nits.ac.in, ankit_rs@ee.nits.ac.in, shilalipi_rs@ee.nits.ac.in, bkr@ee.nits.ac.in

The major drawback of IT-SMC is the singularity issue. The control inputs become excessively high under certain situations. To eliminate singularity issues, non-singular terminal sliding mode control (NT-SMC) is used in [8]. Similarly, non-singular integral terminal sliding mode control (NIT-SMC) is also analyzed in the literature. A new NIT-SMC was reported for robot manipulators in [9] and for quadrotors in [10].

Observers are used to determine unmeasured states or unknown disturbances. In [11], I-SMC inputs are chattering-free, and a super-twisting SMC disturbance observer estimates disturbances was presented. There are recent works on state observer-based I-SMC [12] and disturbance observer-based integral sliding mode control (STO) [13]. Reference [14] discussed a finite-time disturbance observer and I-SMC. In [15], both STO and I-SMC are presented.

This paper proposes an IT-SMC that reaches the origin in a finite time and is robust against unknown matched disturbances. However, there is a singularity issue; the control inputs become unbounded when the state is close to zero. To tackle this problem, we redesigned the sliding surface and proposed a NIT-SMC strategy. An STO is also used to estimate the unmeasured state of the system and an observer-based control is applied to the chaotic Duffing system to track the square and sinusoidal desired trajectories. The main contributions of the paper are given below.

- An IT-SMC design is designed for a class of nonlinear systems.
- The singularity issue in the designed IT-SMC is handled by redesigning the sliding surface and proposing a NIT-SMC strategy.
- STO is designed to estimate the unmeasured states of the system.
- The stability of the proposed observer-based controller is analyzed using the Lyapunov concept, and the performance is tested with the Duffing nonlinear system.

The rest of the paper is arranged in the following manner. After Introduction, Section II describes the problem formulation. Super-twisting observer, IT-SMC, and NIT-SMC are presented in Section III. Tracking the desired trajectories by the Duffing nonlinear system using the proposed observer-based controller and its comparison is shown in Section IV. Finally, this paper is concluded in Section V.

II. Problem formulation

Consider a nonlinear dynamical system in the following manner,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= g + bu + \delta \\ y &= x_1, \end{aligned} \tag{1}$$

where, x_1, x_2 are states of the system, $g(x_1,x_2)$, $b(x_1,x_2)$ are nonlinear functions, y is the output and x_1 is only measurable state. δ is the unknown matched disturbance.

Assumption 1: The nonlinear functions g and b are unknown but bounded by the maximum values g_M and b_M, respectively.

Assumption 2: The matched disturbance δ is unknown but upper bounded by Δ_M.

The objectives of the paper are: (i) To design an STO to observe the unmeasured states of system (1), as in [16] (ii) Using all the states to design an IT-SMC for nonlinear system (1) (iii) To propose NIT-SMC for (1).

III. Design of the observer-based sliding mode controller

A. Super-twisting sliding mode observer design

As STO reported in [1], [17]; here STO-based integral terminal SMC (STO-IT-SMC) and non-singular integral terminal SMC (STO-NIT-SMC) are derived.

The STO dynamics to estimate the states of the system (1) is given in the following form,

$$\begin{aligned} \dot{\hat{x}}_1 &= w_1 + \hat{x}_2 \\ \dot{\hat{x}}_2 &= w_2 + u \end{aligned} \tag{2}$$

where, $\hat{x}_1, \hat{x}_2$ are the observed states of STO. $w_1 = k_1|e_1|^{\frac{1}{2}}\text{sign}(e_1)$ and $w_2 = k_2\text{sign}(e_1)$. The error variables e_1, e_2 are defined as $e_1 = x_1 - \hat{x}_1$ and $e_2 = x_2 - \hat{x}_2$.

Then, the error dynamics is represented as follows,

$$\begin{aligned} \dot{e}_1 &= -k_1|e_1|^{\frac{1}{2}}\text{sign}(e_1) + e_2 \\ \dot{e}_2 &= -k_2\text{sign}(e_1) + \delta(t) \end{aligned} \tag{3}$$

From assumption 2, $|\delta(t)| < \Delta_M$. Choosing the value of $k_1 = 1.5\sqrt{\Delta_M}$ and $k_2 = 1.1\sqrt{\Delta_M}$, results in convergence of error dynamics to zero in finite time. So, errors e_1 and e_2 become zero in finite time $t > T_0$, which is already explained in the literature [16].

Remark 1: The unmeasured state x_2 is observed using the above STO. Accordingly, the IT-SMC and the proposed NIT-SMC are explained in the following subsection.

In the next section, using the above-observed states, we will design IT-SMC and propose an NIT-SMC strategy in the following subsections.

B. The proposed integral terminal SMC

This section will first design the nominal control strategy without disturbance. The nominal system dynamic without disturbance is defined by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= g + bu_{nominal}. \end{aligned} \tag{4}$$

We are introducing a terminal sliding mode in nominal control.

Nominal terminal sliding variable:

$$s = x_2 + px_1^{\alpha};\ 0 < \alpha < 1 \tag{5}$$

with $p > 0$ is a scalar and $\alpha = \frac{a_1}{b_1}$, where a_1 and b_1 are odd positive integer.

Nominal terminal sliding mode control:

$$u_{nominal} = b^{-1}\left[-g - p\alpha x_1^{\alpha-1}x_2 - l_1\ \text{sign}(s)\right] \tag{6}$$

where l_1 is the switching gain. The reachability of nominal dynamics towards the sliding surface s can be ensured by following the Lyapunov analysis. With Lyapunov function $V_n = \frac{1}{2}s^2$, applying (5) and (6), derivative of V_1 obtained as

$$\begin{aligned} \dot{V}_1 &= s\dot{s} \\ &= s\left(\dot{x}_2 + p\alpha x_1^{\alpha-1}\dot{x}_1\right) \\ &= s\left(g + bu_{nominal} + p\alpha x_1^{\alpha-1}x_2\right) \\ &= s\left(-l_1\ \text{sign}(s)\right) \\ &\leq -l_1|s| \end{aligned} \tag{7}$$

where $l_1 > 0$ implies $\dot{V}_1 \leq 0$. When the trajectory reaches s, $s = 0$ and a reduced dynamic is obtained as

$$\begin{aligned} x_2 &= -px_1^{\alpha} \\ \dot{x}_1 &= -px_1^{\alpha}. \end{aligned} \tag{8}$$

Remark 2: The nominal system dynamic (4) converges to the sliding surface (5) using the nominal terminal SMC (6) as proved in (7). From linear dynamics (8), finite-time convergence of x_1, x_2 to the origin is achieved.

Now, for robust control, the integral sliding mode control is implemented to reject disturbances. The integral sliding variable is defined as,

$$\sigma = s_p - s_n \tag{9}$$

where s_p is the terminal sliding of the perturbed system, and s_n is the terminal sliding with the nominal system. Similarly, $\dot{x}_{2p}$ is the perturbed dynamics and $\dot{x}_{2n}$ is the nominal dynamics. Here, the initial value of the perturbed and nominal sliding variables are the same, so that $\sigma = 0$ from the starting point.

Now, consider the total control inputs of the form, $u(t) = u_{nominal} + u_{discon}$ and u_{discon} considered as,

$$u_{discon} = b^{-1}\left[-l_2\ \text{sign}(\sigma)\right] \tag{10}$$

where l_2 is the gain.

The proposed IT-SMC input is given by,

$$u(t) = b^{-1}\left[-g - p\alpha x_1^{\alpha-1}x_2 - l_1\ \text{sign}(s) - l_2\ \text{sign}(\sigma)\right] \tag{11}$$

make the system states converge to zero.

Now, the sliding surface dynamics is

$$\begin{aligned}\dot{\sigma} &= \dot{s}_p - \dot{s}_n \\ &= \dot{x}_{2p} + p\alpha x_1^{\alpha-1} x_2 - \dot{x}_{2n} - p\alpha x_1^{\alpha-1} x_2 \\ &= g + bu(t) + \delta(t) - (g + bu_{nominal}) \\ &= bu_{discon} + \delta(t) \end{aligned} \tag{12}$$

Theorem 1: Let the assumptions 1 and 2 hold. Consider system (1), sliding surface (9) with nominal control (6) and IT-SMC (11) result in convergence of the system states to the equilibrium point.

Proof: Taking the Lyapunov candidate function as follows,

$$\begin{aligned} V_2 &= \frac{1}{2}\sigma^2 \\ \dot{V}_2 &= \sigma\dot{\sigma} \end{aligned} \tag{13}$$

Putting the sliding dynamics (12) and control input (10), in the above equation

$$\begin{aligned}\dot{V}_2 &= \sigma\left[bu_{discon} + \delta(t)\right] \\ &= \sigma\left[-l_2\ sign(\sigma) + \delta(t)\right] \\ &= \sigma\left[\delta(t) - l_2\ sign(\sigma)\right] \\ &= \sigma\left[\Delta_M - l_2\ sign(\sigma)\right] \end{aligned} \tag{14}$$

By choosing the value of $l_2 > \Delta_M$, $\dot{V}_2$ becomes negative definite, and hence confinement of trajectory to the sliding mode is achieved. ■

Remark 3: It is important to note that the control input in (11) contains a singular term. When states cross the x_2 axis, x_1 becomes zero, and control input becomes unbounded. This sets the ground for designing a non-singular integral terminal SMC, as described in the next subsection.

C. The proposed non-singular integral terminal SMC

Redesigning the nominal terminal sliding variable as below,

New nominal terminal sliding variable:

$$s = x_1 + qx_2^{\beta};\ 1 < \beta < 2 \tag{15}$$

with $q > 0$ is a scalar and $\beta = \frac{a_2}{b_2}$, where a_2 and b_2 are odd positive integer.

Nominal non-singular terminal control law:

$$u_{nominal} = b^{-1}\left[-g - \tfrac{1}{q\beta}x_2^{2-\beta} - \bar{l}_1\ \text{sign}(s)\right] \tag{16}$$

with $\bar{l}_1$ is the positive gain.

$$\begin{aligned} V_3 &= \frac{1}{2}s^2 \\ \dot{V}_3 &= s\dot{s} \\ &= s\left(\dot{x}_1 + q\beta x_2^{\beta-1}\dot{x}_2\right) \\ &= s\left(x_2 + q\beta x_2^{\beta-1}(g + bu_{nominal})\right) \\ &= s\left(x_2 + q\beta x_2^{\beta-1}(-\frac{1}{q\beta}x_2^{2-\beta} - \bar{l}_1\ \text{sign}(s))\right) \\ &= s\left(q\beta x_2^{\beta-1}(-\bar{l}_1\ \text{sign}(s))\right) \\ &\leq -\bar{l}_1 q\beta x_2^{\beta-1}|s| \end{aligned} \tag{17}$$

here $\bar{l}_1, q > 0$ are positive constants. From (15), β is defined in such a way that $(\beta - 1) \in (0\ 1)$, $x_2^{\beta-1}$ will be a positive value. Thus in the above inequality, $\dot{V}_3 \leq 0$ is established. When the trajectory is on $s = 0$, the reduced dynamic is obtained as

$$\begin{aligned} x_2 &= \left(\frac{-x_1}{q}\right)^{1/\beta} \\ \dot{x}_1 &= \left(\frac{-x_1}{q}\right)^{1/\beta}. \end{aligned} \tag{18}$$

Remark 4: The nominal terminal sliding surface in (15) is designed in such a way that β makes $\left(\frac{-x_1}{q}\right)^{1/\beta}$ negative, so that x_1, x_2 converge to the origin within a finite time. The nominal system dynamic (4) converges to the sliding surface (15) using the nominal terminal SMC (16) as proved in (17).

Then, for robust control to reject disturbances, the integral sliding variable is defined as

$$\sigma = s_p - s_n \tag{19}$$

Now, consider the integral terminal control inputs of the form, $u = u_{nominal} + u_{discon}$ and u_{discon} considered as,

$$u_{discon} = b^{-1}\left(-\bar{l}_2\ \text{sign}(\sigma)\right) \tag{20}$$

where $\bar{l}_2$ is the switching gain of the robust part of the control. The proposed non-singular integral terminal control input is given by,

$$u = b^{-1}\left[-g - \tfrac{1}{q\beta}x_2^{2-\beta} - \bar{l}_1\ \text{sign}(s) - \bar{l}_2\ \text{sign}(\sigma)\right] \tag{21}$$

make the system states converge to zero.

$$\begin{aligned}\dot{\sigma} &= \dot{s}_p - \dot{s}_n \\ &= x_2 + q\beta x_2^{\beta-1}\dot{x}_{2p} - x_2 - q\alpha x_2^{\beta-1}\dot{x}_{2n} \\ &= q\beta x_2^{\beta-1}(g + bu(t) + \delta(t)) - q\alpha x_2^{\beta-1}(g + bu_{nominal}) \\ &= q\beta x_2^{\beta-1}(bu_{discon} + \delta(t)) \end{aligned} \tag{22}$$

The stability can be proved by following the Theorem.

Theorem 2: Let the assumptions 1 and 2 hold. Consider system (1), sliding surface (15) with nominal control (16) and NIT-SMC (21) result in convergence of the system states to the equilibrium point.

Proof: Taking the Lyapunov candidate function as $V_4 = \frac{1}{2}\sigma^2$. Using (22) and (21), time derivative of $\dot{V}_4$ is obtained as

$$\begin{aligned}\dot{V}_4 &= \sigma\dot{\sigma} \\ &= \sigma\left[q\beta x_2^{\beta-1}(bu_{discon} + \delta(t))\right] \\ &= q\beta x_2^{\beta-1}\left[\sigma(\delta(t) - \bar{l}_2\ sign(\sigma))\right] \\ &= (\Delta_M - \bar{l}_2)q\beta x_2^{\beta-1}|\sigma| \end{aligned} \tag{23}$$

where q, β and $x_2^{\beta-1}$ are positive. Now by choosing, the value of $\bar{l}_2 > g_M + \Delta_M$, the $\dot{V}_4$ becomes negative definite. Hence, the states converge to zero using the proposed controller. ■

The proposed NIT-SMC (21) design can ensure the perturbed system dynamic (1) to behave as nominal system (4) at integral sliding surface σ with robust control (20). Then, on the sliding surface σ, the terminal sliding surface s is defined to get the reduced dynamics (18), which helps for the convergence of x_1, x_2 to the equilibrium point.

IV. Implementation and Results

A nonlinear model of the Duffing nonlinear system is chosen for experimental validation of the proposed controller. The system dynamics of the Duffing nonlinear system ([18], [19]) is as follows,

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.05x_2 + 0.1x_1 - x_1^3 + 7.5cos(t) + u \\ y &= x_1 \end{aligned} \tag{24}$$

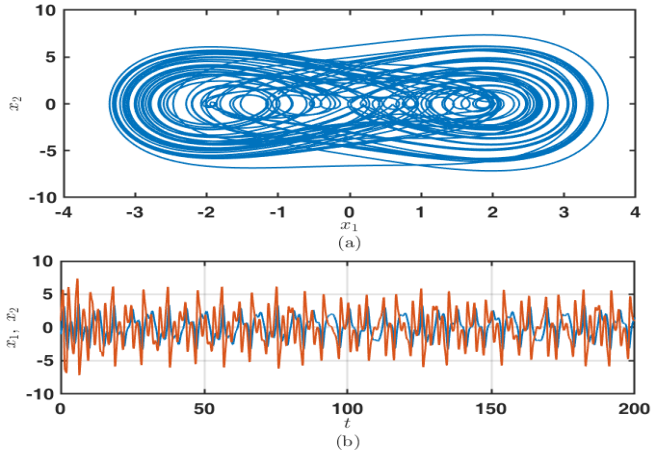


Fig. 1: The Duffing nonlinear system (a) phase space (b)states trajectories.

where $7.5cos(t)$ is unknown matched disturbances. The system behaves chaotically, as shown in Fig. 1. Here, y is the output. The true value of state x_1 is available, whereas state x_2 is not available. Super-twisting observer is used to obtain the value of state x_2 as explained in Section III-A.

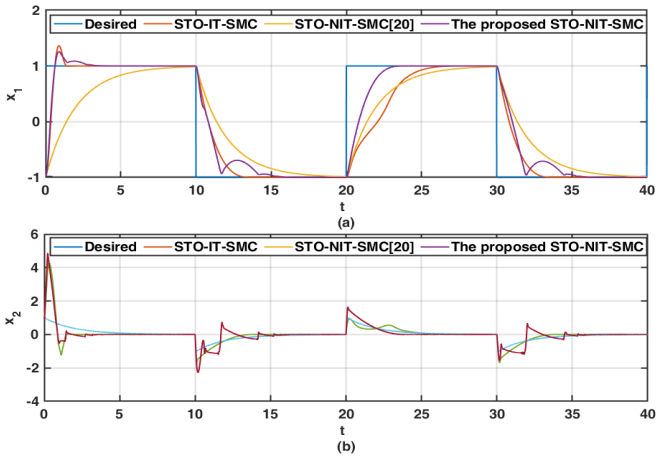


Fig. 2: Tracking a square signal (a) for x_1 state and (b)x_2 state.

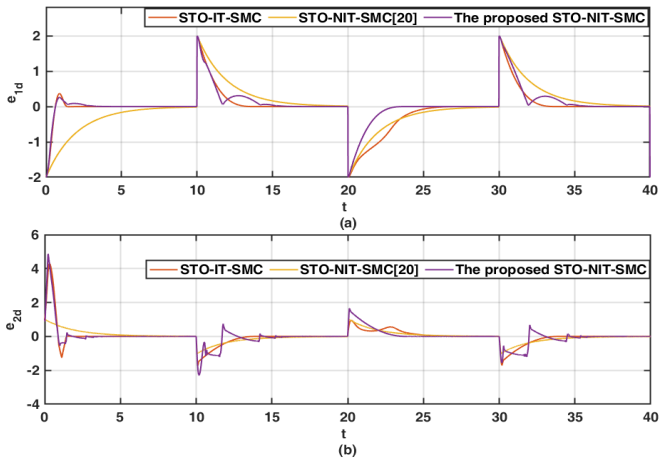


Fig. 3: (a) error state e_{1d} and (b)error state e_{2d} when tracking a square signal.

The super-twisting observer dynamics to estimate the system's states are as follows,

$$\begin{aligned}\dot{\hat{x}}_1 &= k_1|(x_1 - \hat{x}_1)|^{\frac{1}{2}}sign(x_1 - \hat{x}_1) + \hat{x}_2 \\ \dot{\hat{x}}_2 &= k_2 sign(x_1 - \hat{x}_1) + u \end{aligned} \tag{25}$$

To track the desired state x_d, the errors are $e_{1d} = x_1 - x_d$ and $e_{2d} = \hat{x}_2 - \dot{x}_d$. As explained in Section III-B, III-C, the STO-IT-SMC and proposed non-singular STO-NIT-SMC are designed to stabilize $e_{1d}, e_{2d} = 0$ in the following manner.

The terminal sliding variable for STO-IT-SMC:

$$s = e_{2d} + pe_{1d}^{\alpha};\ 0 < \alpha < 1 \tag{26}$$

The terminal sliding variable for STO-NIT-SMC:

$$s = e_{1d} + qe_{2d}^{\beta};\ 1 < \beta < 2 \tag{27}$$

Integral terminal sliding variable: same for both STO-IT-SMC and STO-NIT-SMC,

$$\sigma = s_p - s_n. \tag{28}$$

TABLE I: Total control energy, integral absolute error (IAE) and mean square error (MSE) when different control techniques are applied.

Techniques	Desired signal	Control energy (x10^3)	IAE for e_{1d}	IAE for e_{2d}	MSE for e_{1d}	MSE for e_{2d}
STO-IT-SMC	$square(0.1\pi t)$	187.11	902.64	875.31	0.2713	0.3197
	$sin(0.4\pi t)$	438.41	98.36	275.84	0.0310	0.3248
STO-NIT-SMC [20]	$square(0.1\pi t)$	499.42	1587.3	791.10	0.4024	0.1001
	$sin(0.4\pi t)$	265.30	201.35	102.41	0.0506	0.0129
The proposed STO-NIT-SMC	$square(0.1\pi t)$	**449.92**	**775.99**	968.57	**0.2121**	0.3587
	$sin(0.4\pi t)$	**251.97**	**128.91**	289.78	**0.0359**	0.3052

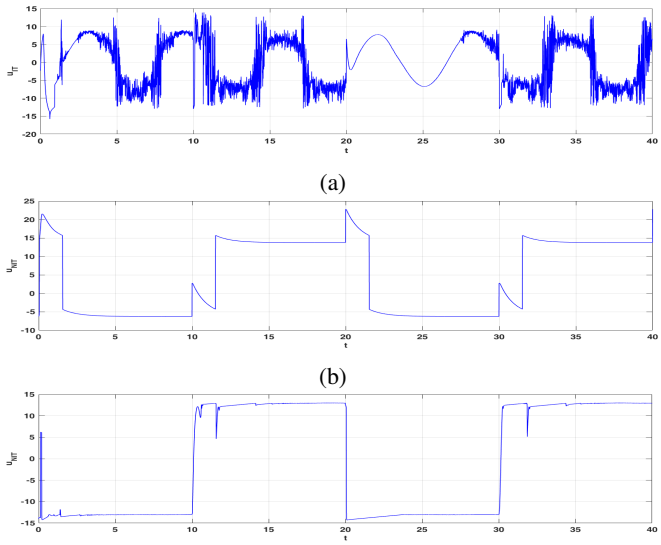


Fig. 4: Control input (a)u_{IT} for STO-IT-SMC, (b)u_{NIT} for the existing STO-NIT-SMC [20] and (b)u_{NIT} for the proposed STO-NIT-SMC when, tracking a square signal.

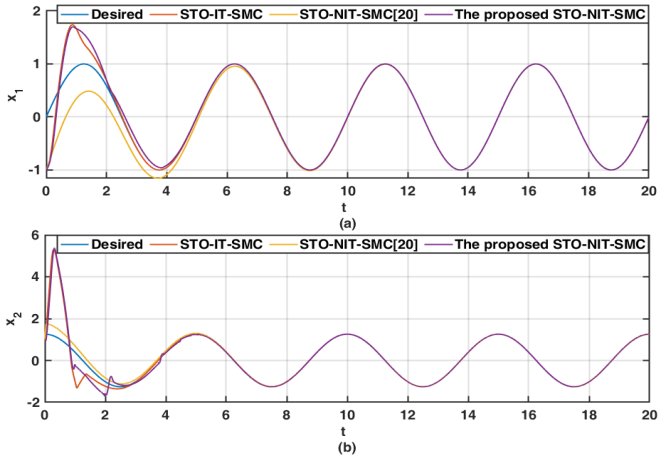


Fig. 5: Tracking a sinusoidal curve (a) for x_1 state and (b)x_2 state.

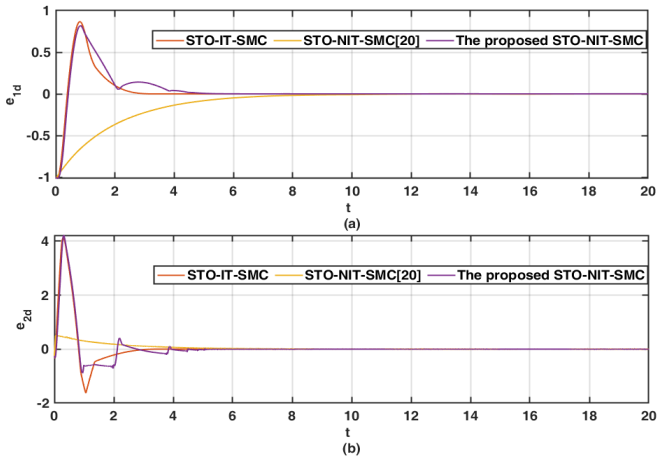


Fig. 6: (a) error state e_{1d} and (b) error state e_{2d} when tracking a sinusoidal signal.

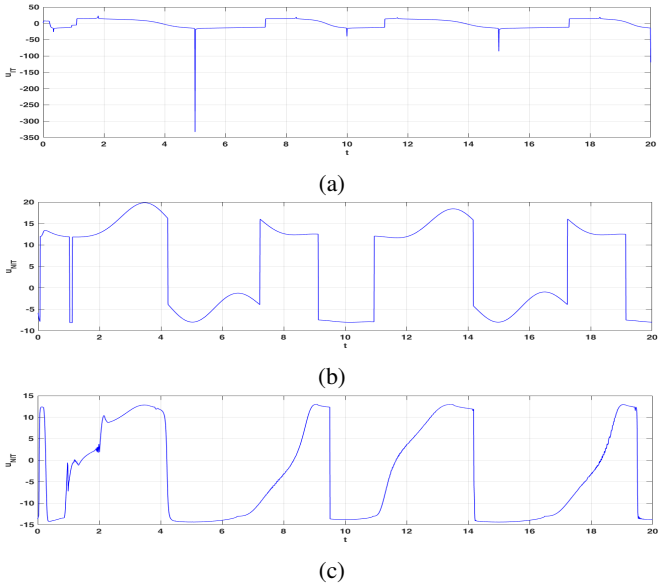


Fig. 7: Control input (a)u_{IT} for STO-IT-SMC , (b)u_{NIT} for the existing STO-NIT-SMC [20] and (b)u_{NIT} for the proposed STO-NIT-SMC when, tracking a sinusoidal curve.

The control input for STO-IT-SMC,

$$u_{IT} = b^{-1}\left[-g - p\alpha e_{1d}^{\alpha-1} e_{2d} - l_1 \text{ sign}(s) - l_2 \text{ sign}(\sigma)\right] \tag{29}$$

The control input for STO-NIT-SMC,

$$u_{NIT} = b^{-1}\left[-g - \tfrac{1}{q\beta} e_{2d}^{2-\beta} - \bar{l}_1 \text{ sign}(s) - \bar{l}_2 \text{ sign}(\sigma)\right] \tag{30}$$

The existing NIT-SMC is considered as presented in [20], and the STO-based existing NIT-SMC is compared with the STO-based proposed NIT-SMC.

Using (29) and (30), the Duffing chaotic system tracks the desired square signal and desired sine signal. Fig. 2 shows states of the system tracking the desired square signal, the error between the system states and desired states are shown in Fig. 3 and the respective control inputs u_{IT}, the existing u_{NIT} and the proposed u_{NIT} are shown in Fig. 4(a-c), respectively. A square signal is a non-smooth signal. The proposed controller STO-NIT-SMC achieves quicker convergence than STO-IT-SMC and the existing STO-NIT-SMC. Further, in Fig. 4, it is observed that the chattering phenomenon is not present in the control input u_{NIT} in Fig. 4(b) and in Fig. 4(c), unlike the control input u_{IT} in Fig. 4(a). Hence, the proposed controller can be preferred for practical applications.

Furthermore, tracking of a sinusoidal signal is demonstrated in Fig. 5. The error between the system states and the desired state is shown in Fig. 6. The control inputs u_{IT} the existing u_{NIT} and the proposed u_{NIT} are shown in Fig. 7. When IT-SMC tracks the sinusoidal signal, the singularity issue is visible in Fig. 7(a). However, when the same sinusoidal signal is tracked by the existing NIT-SMC [20] as in Fig. 7(b) and the proposed NIT-SMC as in Fig. 7(c), the singularity issue is eliminated. This is one more advantage of using the proposed controller over the IT-SMC.

As shown in the comparison Table I, The proposed controller achieves lesser control effort, integral absolute error and mean square error for the desired signal as compared to both STO-IT-SMC and STO-NIT-SMC [20].

V. Conclusion

This paper proposes a super-twisting observer-based nonsingular integral terminal sliding mode controller for a class of nonlinear systems. The Duffing chaotic system is considered for testing the performance of the proposed controller. The successful tracking of the desired square and sinusoidal signals is shown using the proposed STO-NIT-SMC. A performance comparison is shown among STO-IT-SMC, the existing STO-NIT-SMC and the proposed STO-NIT-SMC regarding control input, singularity issue, and chattering. The proposed controller outperforms its competitor. Further, this proposed control approach can be used for chaos suppression and chaos synchronization, such as vibration control, secure communication, etc.

References

[1] C. Edwards and S. K. Spurgeon, "On the development of discontinuous observers," *International Journal of control*, vol. 59, no. 5, pp. 1211–1229, 1994.

[2] M. Rubagotti, A. Estrada, F. Castaños, A. Ferrara, and L. Fridman, "Integral sliding mode control for nonlinear systems with matched and unmatched perturbations," *IEEE Transactions on Automatic Control*, vol. 56, no. 11, pp. 2699–2704, 2011.

[3] Y. M. Sam, J. H. Osman, and M. R. A. Ghani, "A class of proportional-integral sliding mode control with application to active suspension system," *Systems & control letters*, vol. 51, no. 3-4, pp. 217–223, 2004.

[4] P. Roy and B. K. Roy, "Sliding mode control versus fractional-order sliding mode control: Applied to a magnetic levitation system," *Journal of Control, Automation and Electrical Systems*, vol. 31, pp. 597–606, 2020.

[5] M. Labbadi and M. Cherkaoui, "Robust integral terminal sliding mode control for quadrotor uav with external disturbances," *International Journal of Aerospace Engineering*, vol. 2019, 2019.

[6] A. Riani, T. Madani, A. Benallegue, and K. Djouani, "Adaptive integral terminal sliding mode control for upper-limb rehabilitation exoskeleton," *Control Engineering Practice*, vol. 75, pp. 108–117, 2018.

[7] H. Wang, Z. Li, X. Jin, Y. Huang, H. Kong, M. Yu, Z. Ping, and Z. Sun, "Adaptive integral terminal sliding mode control for automobile electronic throttle via an uncertainty observer and experimental validation," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 9, pp. 8129–8143, 2018.

[8] S. Yi and J. Zhai, "Adaptive second-order fast nonsingular terminal sliding mode control for robotic manipulators," *ISA transactions*, vol. 90, pp. 41–51, 2019.

[9] Y. Su and C. Zheng, "A new nonsingular integral terminal sliding mode control for robot manipulators," *International Journal of Systems Science*, vol. 51, no. 8, pp. 1418–1428, 2020.

[10] M. Labbadi, M. Djemai, and S. Boubaker, "A novel non-singular terminal sliding mode control combined with integral sliding surface for perturbed quadrotor," *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, vol. 236, no. 5, pp. 999–1009, 2022.

[11] A. Chalanga, S. Kamal, and B. Bandyopadhyay, "Continuous integral sliding mode control: A chattering free approach," in *2013 IEEE International Symposium on Industrial Electronics*, pp. 1–6, IEEE, 2013.

[12] R. Cui, L. Chen, C. Yang, and M. Chen, "Extended state observer-based integral sliding mode control for an underwater robot with unknown disturbances and uncertain nonlinearities," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 8, pp. 6785–6795, 2017.

[13] J. Wang, F. Wang, X. Wang, and L. Yu, "Disturbance observer based integral terminal sliding mode control for permanent magnet synchronous motor system," *Kybernetika*, vol. 55, no. 3, pp. 586–603, 2019.

[14] B. Li, Q. Hu, Y. Yang, and O. A. Postolache, "Finite-time disturbance observer based integral sliding mode control for attitude stabilisation under actuator failure," *IET control theory & applications*, vol. 13, no. 1, pp. 50–58, 2019.

[15] S. K. Pandey, S. L. Patil, U. M. Chaskar, and S. Phadke, "State and disturbance observer-based integral sliding mode controlled boost dc–dc converters," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 66, no. 9, pp. 1567–1571, 2018.

[16] J. Davila, L. Fridman, and A. Levant, "Second-order sliding-mode observer for mechanical systems," *IEEE transactions on automatic control*, vol. 50, no. 11, pp. 1785–1789, 2005.

[17] A. Chalanga, S. Kamal, L. M. Fridman, B. Bandyopadhyay, and J. A. Moreno, "Implementation of super-twisting control: Super-twisting and higher order sliding-mode observer-based approaches," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 6, pp. 3677–3685, 2016.

[18] E. Tamaševičiūtė, A. V. Tamaševičius, G. Mykolaitis, S. Bumelienė, and E. Lindberg, "Analogue electrical circuit for simulation of the duffing-holmes equation," *Nonlinear Analysis: Modelling and Control*, vol. 13, no. 2, pp. 241–252, 2008.

[19] H. Zhao, Y. Lin, and Y. Dai, "Hidden attractors and dynamics of a general autonomous van der pol–duffing oscillator," *International Journal of Bifurcation and Chaos*, vol. 24, no. 06, p. 1450080, 2014.

[20] R. M. Asl, Y. S. Hagh, R. Palm, and H. Handroos, "Integral non-singular terminal sliding mode controller for nth-order nonlinear systems," *IEEE Access*, vol. 7, pp. 102792–102802, 2019.